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Counterintuitive Effects of Excluding Cross-Loadings on Overall Model Fit

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ABSTRACT
In structural equation modeling or factor analysis, excluding nonzero cross-loadings is generally regarded as a form of model misspecification. It is also commonly believed that excluding a greater number or larger size of cross-loadings leads to a more severe model misspecification, thereby resulting in a poorer model-fit. However, a special form of cross-loadings in the population can be perfectly accounted for by the structural model without causing any model misspecification. By exploring this form, this article shows a paradoxical phenomenon in which a model that excludes greater number or larger size of cross-loadings may, counterintuitively, exhibit better fit. Analysis and numerical examples are used to illustrate and support the findings.

KEYWORDS
Cross-loading; model-fit; model misspecification; Monte Carlo; rotation

1. Introduction

Structural equation modeling (SEM) is a multivariate statistical method to analyze relationships among latent and manifest variables, widely used in social and behavioral research (Bollen, 1989; Brown, 2015; Kline, 2023). Unidimensionality is a desirable property in scale development and is strongly recommended in SEM (Anderson & Gerbing, 1988). Under this property, the specified model does not have cross-loadings. However, such a model may not fit the data well. Typically, model-fit becomes worse as more or larger cross-loadings are excluded. This article reveals a paradoxical phenomenon whereby model-fit improves as greater number or larger size of cross-loadings are excluded from the model.

Cross-loadings pose both theoretical and practical challenges in SEM. On one hand, they reflect the complexity of indicator-construct relationships in the real-world. On the other hand, they increase the difficulty of model interpretation and blur the conceptual boundaries between latent factors, thereby compromising theoretical clarity. To simplify model interpretation, items with cross-loadings are reworded or removed in the stage of exploratory factor analysis (EFA). As a result, confirmatory factor analysis (CFA) and SEM typically adopt a simple measurement model in which each manifest variable loads on only one latent factor (Jöreskog, 1969, 1971). Although this assumption is theoretically appealing, excluding nonzero cross-loadings can result in severe model misspecification and biased parameter estimates (Bentler & Chou, 1987; Sörbom, 1989; Stromeyer et al., 2015; Yuan et al., 2003, 2008). In operation, applied researchers often employ models with cross-loadings to better fit the data. Advices on including cross-loadings have been given for specific designs and data/model structure (see e.g., Morin, 2013). These considerations highlight the need for a more nuanced understanding of cross-loadings and their implications for model specification.

Substantively, excluding nonzero cross-loadings has two consequences. First, it may bias the estimates of existing factor loadings, preventing them from accurately reflecting the true contribution of each manifest variable to its latent factor (Hsu et al., 2014; Wei et al., 2022; Ximénez et al., 2022; Yuan et al., 2008). Second, the specified model often compensates for the excluded cross-loadings by inflating/deflating the correlations between factors, causing misleading interpretations of the relationships among factors (Asparouhov & Muthén, 2009; MacCallum et al., 1992; Yuan et al., 2003). In addition, excluding nonzero cross-loadings alters the model-implied covariance matrix, which negatively affects overall model-fit (Li et al., 2020; Morin et al., 2016). Although Cao and Liang (2024) found that excluding small cross-loadings may have a negligible effect on fit indices, it is generally assumed that model-fit deteriorates as the number or size of excluded cross-loadings increases.

This study aims to clarify a seemingly counterintuitive phenomenon, in which model-fit improves as more or larger cross-loadings in the population are not accounted for by a SEM model. We first introduce a special form of cross-loadings whose exclusion does not cause any problems in reproducing the population covariance matrix. Building on this, we demonstrate that the specified model tends to achieve a better fit when a greater number or larger size of cross-loadings in the population are left unaccounted for. The finding contradicts the conventional view that a greater extent of model misspecification leads to a poorer model-fit. The article continues to explore the mechanism for this phenomenon and elaborate on the meaning of goodness-of-fit. Throughout the article, we will use the normal-distribution-based discrepancy function F_{ML} to measure the goodness-of-fit.

Section 2 introduces a special form of cross-loadings that can be perfectly incorporated into the factor structure without altering the population covariance matrix, followed by an analysis on the conditions for such a result to hold or not to hold as well as the corresponding consequences. Section 3 presents numerical examples to illustrate the consequence of goodness-of-fit when cross-loadings are excluded. Section 4 uses Monte Carlo simulation to further demonstrate that better model-fit can be achieved when a greater number or larger size of cross-loadings in the population are excluded. The article concludes with a discussion of the theoretical and practical implications of our findings.

2. Cross-Loading Transformation and Its Implication

Factor rotation is commonly used to simplify the factor loading matrix in EFA (Carroll, 1953; Harman, 1976; Jennrich & Sampson, 1966; Kaiser, 1958). In this section, we first introduce a pattern of cross-loadings that can be transformed to a unidimensional loading matrix via factor rotation without altering the population covariance matrix, then show the different consequences of excluding such cross-loadings in saturated and unsaturated structural models.

2.1. Transformation of Cross-Loadings

Suppose p manifest variables consist of q blocks $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_q$, each block targets one of the q factors $f_1, f_2, \ldots, f_q$, respectively. The block $\mathbf{x}_k$ has p_k manifest variables, $k = 1, 2, \ldots, q$. Let $\mathbf{x} = (\mathbf{x}_1', \mathbf{x}_2', \ldots, \mathbf{x}_q')'$, $\boldsymbol{f} = (f_1, f_2, \ldots, f_q)'$ with $\mathrm{Cov}(\boldsymbol{f}) = \Phi_f$, and $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1', \boldsymbol{\varepsilon}_2', \ldots, \boldsymbol{\varepsilon}_q')'$ with $\mathrm{Cov}(\boldsymbol{\varepsilon}) = \Psi$.

In the population, the block of indicators $\mathbf{x}_k$ not only loads on the target factor f_k with a loading vector $\boldsymbol{\lambda}_k$ but is also associated with the non-target factor f_j by a cross-loading vector γ_{kj}, $j = 1, 2, \ldots, q$. The relationships between variables and factors can be expressed as

$$\mathbf{x} = \mathbf{A}\boldsymbol{f} + \boldsymbol{\varepsilon} \tag{1}$$

with a population covariance matrix

$$\Sigma = \mathbf{A}\Phi_f\mathbf{A}' + \Psi, \tag{2}$$

where

$$\mathbf{A} = \begin{pmatrix} \boldsymbol{\lambda}_1 & \gamma_{12} & \cdots & \gamma_{1q} \\ \gamma_{21} & \boldsymbol{\lambda}_2 & \cdots & \gamma_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{q1} & \gamma_{q2} & \cdots & \boldsymbol{\lambda}_q \end{pmatrix}.$$

Note that both $\boldsymbol{\lambda}_k$ and γ_{kj} are column vectors of length p_k.

Suppose γ_{kj} and $\boldsymbol{\lambda}_k$ are proportional with

$$\gamma_{kj} = c_{kj}\boldsymbol{\lambda}_k, \tag{3}$$

where c_{kj} is a scalar, and the entries c_{kj} form a $q \times q$ matrix $\mathbf{C} = (c_{kj})$. Then we can write the loading matrix $\mathbf{A}$ as $\mathbf{A} = \Lambda\mathbf{C}$, where $\Lambda = \mathrm{diag}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \ldots, \boldsymbol{\lambda}_q)$ is block diagonal. Therefore, Equation (1) can be written as

$$\mathbf{x} = \Lambda\mathbf{C}\boldsymbol{f} + \boldsymbol{\varepsilon} = \Lambda\boldsymbol{\xi} + \boldsymbol{\varepsilon}, \tag{4}$$

where $\xi = \mathbf{C}\boldsymbol{f}$ represents a vector of new factors obtained via a linear transformation of $\boldsymbol{f}$ with

$$\mathrm{Cov}(\boldsymbol{\xi}) = \mathbf{C}\Phi_f\mathbf{C}' = \Phi_\xi. \tag{5}$$

Equation (4) implies that rotation $\mathbf{C}$ transforms $\mathbf{A}$ with cross-loadings into Λ without cross-loadings, and there exists

$$\Sigma = \Lambda\Phi_\xi\Lambda' + \Psi. \tag{6}$$

That is, all cross-loadings can be absorbed by the covariance matrix Φ_ξ without affecting the model-implied covariance matrix Σ.

2.2. Cross-Loadings with Saturated and Unsaturated Structural Models

The cross-loading-rotation pattern for the factor model in Equation (4) equally applies to SEM model, but with conditions. Note that a general SEM model consists of a measurement model and a structural model (Jöreskog & Sörbom, 1996). The structural model can be further classified as either saturated or unsaturated. Two latent variables are directly connected if they have a one-way predictive relationship, a two-way (reciprocal) predictive relationship, or a correlation relationship. A structural model is saturated if all the latent variables are directly connected. In addition to being directly connected, parameters of a saturated structural model are not subject to any constraints. The structural model is unsaturated if any two latent variables are not directly connected or the parameters of the structural model are subject to constraints.

The rotational transformation of cross-loadings described in Section 2.1 behaves differently in saturated and unsaturated structural models. Specifically, in a saturated structural model, cross-loadings that satisfy Equation (3) do not cause any misspecifications when excluded by a unidimensional measurement model, all the associations among variables due to cross-loadings can be picked up by the structural model. However, in an unsaturated structural model, the cross-loadings cannot be perfectly absorbed by Φ_ξ. This is because, due to either parameter constraints or not direct connections of latent variables, the extra relationships among the observed variables via the extra loadings cannot be recovered by the covariance matrix Φ_ξ. We have the following result.

Lemma 1. Excluding cross-loadings proportional to Equation (3) does not cause any model misspecifications if the structural model is saturated.

Of course, if the c_{kj} in Equation (3) satisfies certain conditions, e.g., only a few of them are nonzero, then the structural model does not need to be saturated for Φ_ξ to fully pick up the associations among the indicators under a unidimensional model, as demonstrated by the following example.

Example 1. This example is based on a three-factor model as shown in Figure 1, where each factor is targeted by 3

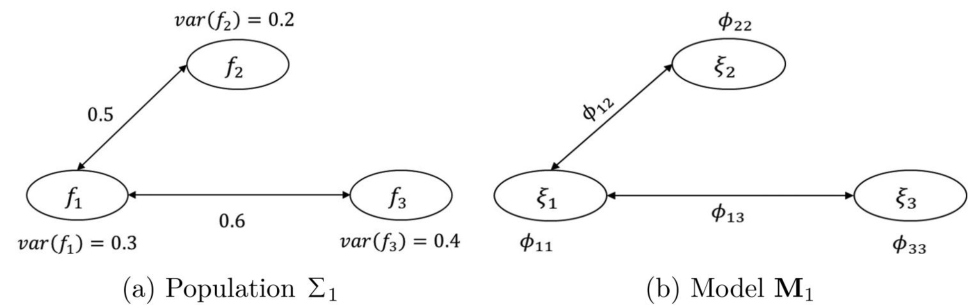


Figure 1. The structural model of Σ_1 and $\mathbf{M}_1$ for Example 1.

indicators. In the population, only the indicators in $\mathbf{x}_1$ cross-load on f_2 and f_3 with the loading matrix

$$\mathbf{A} = \begin{pmatrix} 1.00 & 0.60 & 0.80 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.50 & 0.30 & 0.40 & 1.00 & 0.40 & 0.80 & 0 & 0 & 0 \\ 0.30 & 0.18 & 0.24 & 0 & 0 & 0 & 1.00 & 0.40 & 0.60 \end{pmatrix}', \tag{7}$$

where the cross-loadings satisfy Equation (3). According to the path diagram in Figure 1a, the covariance matrix of $\boldsymbol{f} = (f_1, f_2, f_3)'$ is

$$\Phi_f = \begin{pmatrix} 0.3 & 0.5 & 0.6 \\ 0.5 & 0.2 & 0 \\ 0.6 & 0 & 0.4 \end{pmatrix}.$$

In addition, the variance matrix of measurement errors is $\boldsymbol{\Psi} = 0.2\mathbf{I}_9$, where $\mathbf{I}_k$ denote a k-dimensional identity matrix. The population covariance matrix $\boldsymbol{\Sigma}_1$ of the nine indicators is obtained by substituting the above $\mathbf{A}$, Φ_f, and $\boldsymbol{\Psi}$ into Equation (2). The model $\mathbf{M}_1 = \mathbf{M}(\boldsymbol{\theta}_1)$ is implied by the path diagram in Figure 1b together with a unidimensional measurement model, where the structural model is unsaturated.

When $\boldsymbol{\Sigma}_1$ is fitted by $\mathbf{M}_1$, all cross-loadings are fully absorbed by the structural model with a factor covariance matrix

$$\Phi_\xi = \begin{pmatrix} 1.246 & 0.600 & 0.720 \\ 0.600 & 0.200 & 0 \\ 0.720 & 0 & 0.400 \end{pmatrix},$$

where the values can be obtained analytically by applying the standard rules of matrix multiplication according to Equation (5). Equation (6) holds because the cross-loadings of $\mathbf{x}_1$ on f_2 and f_3 are transformed onto $var(\xi_1)$, $cov(\xi_1, \xi_2)$, and $cov(\xi_1, \xi_3)$, as shown by the inflated elements ϕ_{11}, ϕ_{12}, and ϕ_{13} of Φ_ξ. Whereas the parameters ϕ_{22}, ϕ_{33}, and ϕ_{23} remain the same between Φ_f and Φ_ξ. Therefore, the unsaturated structural model in Figure 1b is sufficient to account for the cross-loadings in Equation (7).

2.3. Cross-Loadings Not Proportional to Target Loadings

This subsection further explores the situation where the cross-loadings do not follow the proportionality in Equation (3), while the structural model continues to be saturated.

For simplicity, we consider the first block $\mathbf{x}_1$ with p_1 indicators. Within this block, the target loadings of $\mathbf{x}_1$ on f_1 are $\lambda_{11}, \lambda_{21}, \ldots, \lambda_{p_1 1}$, and the cross-loadings of $\mathbf{x}_1$ on f_2 are $\gamma_{12}, \gamma_{22}, \ldots, \gamma_{p_1 2}$, where $\gamma_{l2} = c_l \lambda_{l1}$, $l = 1, 2, \ldots, p_1$. The indicators in $\mathbf{x}_1$ do not load on factors $f_3, f_4, \ldots, f_q$. Indicators in $\mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_q$ only load on their target factors. Suppose the above population is fitted by a unidimensional model. We have the following results.

Lemma 2. (a) The value of F_{ML} decreases as d increases from 1 to p_1 if $c_1 = c_2 = \cdots = c_d = c$ and $c_{d+1} = c_{d+2} = \cdots = c_{p_1} = 0$, where c is a nonzero constant.
(b) Let $c_2 = c_3 = \cdots = c_{p_1} = c > 0$ be fixed while c_1 varies. Then, the discrepancy function F_{ML} decreases to 0 as c_1 increases from 0 to c, and then starts to increase as c_1 continues to increase.

Lemma 2(a) indicates that model-fit improves as more cross-loadings are excluded. This is because the structural model is capable of absorbing more cross-loadings as d increases from 0 to p_1. When $d = p_1$, the situation becomes that of Lemma 1, where all cross-loadings are absorbed by the structural model without creating any model misspecification. Lemma 2(b) reveals a non-monotonic trend in model-fit as the size of excluded cross-loadings increases. The model-fit gradually improves as c_1 increases from 0 to c, and starts to deteriorate as c_1 continues to increase. Sections 3 and 4 illustrate the results of Lemmas 1 and 2 numerically and by Monte Carlo.

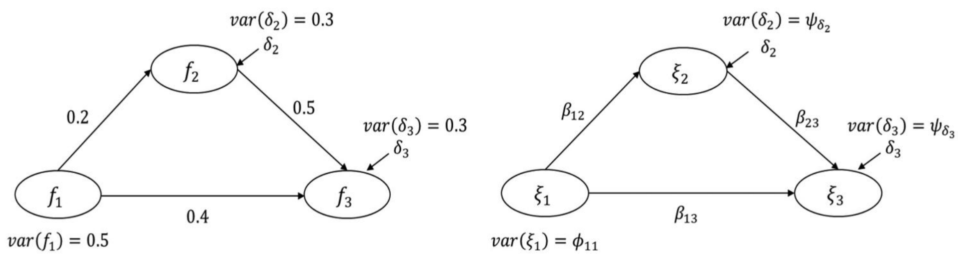
3. Illustrations

This section illustrates the effect of excluding cross-loadings on model-fit using numerical examples. In each example, the population covariance matrix Σ is specified using Equations (1) and (2). The model $\mathbf{M} = \mathbf{M}(\boldsymbol{\theta})$ is derived from a unidimensional measurement model, as specified in Equation (4), where $\boldsymbol{\theta}$ is the parameter vector. The normal-distribution-based maximum likelihood (NML) (Jöreskog, 1969) is then used to obtain the population value F_{ML0} by minimizing the discrepancy function

$$F_{ML}[\Sigma, \mathbf{M}(\boldsymbol{\theta})] = \operatorname{tr}\left[\Sigma \mathbf{M}^{-1}(\boldsymbol{\theta})\right] - \log|\Sigma \mathbf{M}^{-1}(\boldsymbol{\theta})| - p, \tag{8}$$

where the computation is carried out using the Fisher-scoring algorithm (Yuan & Bentler, 2017) and implemented in MATLAB R2020a[1] (The MathWorks Inc, 2020).

[1]Interested readers can contact the authors of the article for the Matlab code.



(a) Population Σ_{2h} (b) Model $\mathbf{M}_2$

Figure 2. The structural model of Σ_{2h} and $\mathbf{M}_2$ for Example 2.

Table 1. The value of F_{ML0} for $\mathbf{M}_2$ as h varies in Example 2.

h	0	0.2	0.4	0.6	0.8	1.0
F_{ML0}	0	0	0	0	0	0

Example 2. This example is based on a latent-variable mediation model, as shown in Figure 2, where each latent variable is measured by 3 indicators. In the population, the loading matrices are designed as

$$\mathbf{A}_h = \begin{pmatrix} 1 & 0.6 & 0.8 & 0.5h & 0.2h & 0.4h & h & 0.4h & 0.6h \\ h & 0.6h & 0.8h & 1 & 0.4 & 0.8 & 0.5h & 0.2h & 0.3h \\ 0.5h & 0.3h & 0.4h & h & 0.4h & 0.8h & 1 & 0.4 & 0.6 \end{pmatrix}',$$

where h is a constant with values 0, 0.2, 0.4, 0.6, 0.8, and 1.0, respectively. Clearly, the cross-loadings satisfy the proportional relationship specified in Equation (3).

According to the values of the parameters given in Figure 2a, the factor covariance matrix is

$$\Phi_f = \begin{pmatrix} 0.50 & 0.10 & 0.25 \\ 0.10 & 0.32 & 0.20 \\ 0.25 & 0.20 & 0.50 \end{pmatrix}.$$

In addition, the variance matrix of measurement error is set as $\Psi = 0.1\mathbf{I}_9$. The population covariance matrix Σ_{2h} at each h is obtained by substituting the above $\mathbf{A}_h$, Φ_f, and Ψ into Equation (2). The model $\mathbf{M}_2 = \mathbf{M}(\boldsymbol{\theta}_2)$ is implied by the path diagram in Figure 2b for the structural model together with a unidimensional measurement model.

Each Σ_{2h} is then fitted by $\mathbf{M}_2$ and the value of the resulting F_{ML0} is given in Table 1, which shows that F_{ML0} remains zero as h varies. This indicates that the population with proportional cross-loadings and a saturated structural model can be perfectly accounted for by a unidimensional measurement model together with a saturated structural model.

The next example shows that if the structural model is unsaturated, the model cannot pick up all the associations among the indicators due to cross-loadings even if the population Φ_f is generated by the same unsaturated structural model.

Example 3. The loading matrix $\mathbf{A}_h$ in Example 3 is the same as in Example 2 with six values of h. However, Example 3 adopts an unsaturated structure model, in which there is no direct path from f_1 to f_3, meaning a complete mediation structure, as shown in Figure 3. According to the parameter specifications in Figure 3a, the factor covariance matrix is

$$\Phi_f = \begin{pmatrix} 0.50 & 0.10 & 0.05 \\ 0.10 & 0.32 & 0.16 \\ 0.05 & 0.16 & 0.38 \end{pmatrix}.$$

In addition, the variance matrix of measurement error is set as $\Psi = 0.1\mathbf{I}_9$. Consequently, the population covariance matrix Σ_{3h} can be derived by Equation (2). Model $\mathbf{M}_3 = \mathbf{M}(\boldsymbol{\theta}_3)$ has a unidimensional measurement model and its structural model is given by the path diagram in Figure 3b.

We fit each Σ_{3h} by $\mathbf{M}_3$ and $\mathbf{M}_2$ sequentially, and the results of F_{ML0} are reported in Table 2. When using $\mathbf{M}_3$ to fit Σ_{3h}, $F_{ML0} > 0$ (although small) whenever $h > 0$, meaning that cross-loadings cannot be perfectly absorbed by the structural model. This is because when there is no direct path from ξ_1 to ξ_3, the resulting Φ_ξ is unable to pick up all the extra association created by the cross-loadings. However, when Σ_{3h} is fitted by $\mathbf{M}_2$, the values of F_{ML0} remain zero across all values of h, indicating that all cross-loadings are perfectly accounted for. Therefore, a saturated structural model is the key.

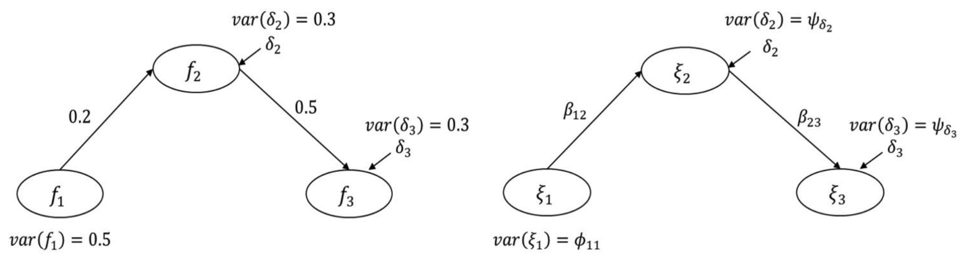
Example 3 shows that an unsaturated structural model may not have the mechanism to fully compensate for the association left out by cross-loadings even if the cross-loadings are proportional to the target loadings. The following example further illustrates that excluding a greater number or larger size of cross-loadings may paradoxically result in a better model-fit.

Example 4. This example is based on a three-factor model, where f_1 is targeted by indicators x_1, x_2, x_3, x_4, x_5; f_2 is targeted by indicators x_6, x_7, x_8; and f_3 is targeted by indicators x_9, x_{10}, x_{11}. In the population, indicators x_1, x_2, x_3, x_4, x_5 not only load on f_1 but also cross-load on f_2. For analytical clarity, no other cross-loadings exist.

Let $\mathbf{1}_k$ and $\mathbf{0}_k$ represent k-dimensional column vectors with elements 1 and 0, respectively. The loading matrix of Example 4 is

$$\mathbf{A}_h = \begin{pmatrix} 0.5\mathbf{1}_5 & 0.5\mathbf{c} & \mathbf{0}_5 \\ \mathbf{0}_3 & 0.7\mathbf{1}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & 0.9\mathbf{1}_3 \end{pmatrix}, \quad (9)$$

where $\mathbf{c} = (c_1, c_2, c_3, c_4, c_5)'$ represents the ratios of the cross-loadings to their target loadings. Note that, unless all the



(a) Population Σ_{3h} (b) Model $\mathbf{M}_3$

Figure 3. The structural model of Σ_{3h} and $\mathbf{M}_3$ for Example 3.

Table 2. The values of F_{ML0} for $\mathbf{M}_3$ and $\mathbf{M}_2$ as h varies in Example 3.

h		0	0.2	0.4	0.6	0.8	1.0
F_{ML0}	$\mathbf{M}_3$	0	0.0203	0.0582	0.1075	0.1664	0.2237
	$\mathbf{M}_2$	0	0	0	0	0	0

Table 3. Eight conditions of **c** that are not proportional to the target loadings.

ID	$\mathbf{c} = (c_1, c_2, c_3, c_4, c_5)'$	
cond 1	$c_1 = c_2 = c_3 = c_4 = c_5 = h$	
cond 2	$c_1 = c_2 = c_3 = c_4 = h,$	$c_5 = 0$
cond 3	$c_1 = c_2 = c_3 = h,$	$c_4 = c_5 = 0$
cond 4	$c_1 = c_2 = h,$	$c_3 = c_4 = c_5 = 0$
cond 5	$c_1 = h,$	$c_2 = c_3 = c_4 = c_5 = 0$
cond 6	$c_1 = h,$	$c_2 = c_3 = c_4 = c_5 = 0.2$
cond 7	$c_1 = h,\ c_2 = c_3 = 0.2,\ c_4 = c_5 = 0.4$	
cond 8	$c_1 = h,\ c_2 = 0.2,\ c_3 = 0.4,\ c_4 = 0.6,\ c_5 = 0.8$	

elements of **c** are equal, the cross-loadings in Equation (9) do not satisfy the condition in Equation (3). We have 8 different conditions on **c**, as given in Table 3, where h is a constant ranging from 0 to 1.5 in steps of 0.01, with 151 distinct values. Conditions 1 to 5, progressively reducing the number of cross-loadings, are designed to explore how model-fit changes as the number of excluded cross-loadings varies. Conditions 6 to 8 are intended to explore how model-fit changes as the size of the excluded cross-loadings varies, where one varying cross-loading and four invariant cross-loadings are chosen.

In addition, the covariance matrix of $\boldsymbol{f}$ is set as

$$\Phi_f = \begin{pmatrix} 1.0 & 0.3 & 0.5 \\ 0.3 & 1.0 & 0.7 \\ 0.5 & 0.7 & 1.0 \end{pmatrix},$$

and the variance matrix of measurement errors is $\boldsymbol{\Psi} = 0.2\mathbf{I}_{11}$. Note that there are 151 values of h in each condition of **c**. For each of the 8×151 combinations of **c** and h, the corresponding population covariance matrix Σ_{4h} is derived using Equation (2). We then fit each Σ_{4h} by a unidimensional three-factor CFA model with freely correlated factors ($\mathbf{M}_4$). For each condition of **c**, 151 values of F_{ML0} are obtained corresponding to the 151 values of h. The plots of the value of F_{ML0} against that of h are shown in Figure 4.

Figure 4a depicts the changes of F_{ML0} for conditions 1 to 5. Condition 1 is the baseline where the value of F_{ML0} remains at 0 as h changes from 0 to 1.5. The number of nonzero cross-loadings in the population is progressively reduced from condition 2 to condition 5. However, in Figure 4a, there is always a value of h at which F_{ML0} becomes smaller as more cross-loadings are excluded. This is because cross-loadings are more likely to be absorbed by the structural model as their number approaches five, as implied in Lemma 2(a).

Figure 4b presents the changes of F_{ML0} for conditions 6 to 8. In each condition, as h increases, the value of F_{ML0} first decreases and then increases. That is, the model-fit first improves and then deteriorates as the size of the excluded cross-loadings increases. The pattern implies that the structural model initially accounts for the cross-loadings more effectively as h increases. But its capacity to do so declines beyond a certain point, as characterized by Lemma 2(b). For example, in condition 6, the structural model cannot account for all the cross-loadings at $h = 0$ and it accounts for an increasing size of cross-loadings as h increases from 0 to 0.2 with $F_{ML0} = 0$ at $h = 0.2$. Then, F_{ML0} starts to increase as h continues to increase.

4. Monte Carlo Results

This section uses Monte Carlo (MC) simulation to show the changes in model-fit as the number and size of excluded cross-loadings vary.

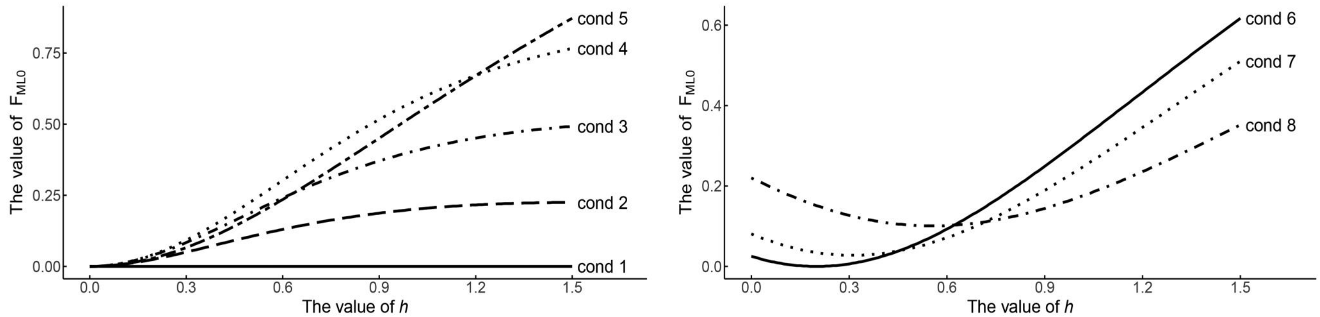
4.1. Design

The populations of the MC study are based on the conditions of Example 4 and so is the model specification, where $8 \times 151 = 1208$ conditions of **c** and h are presented.

For each of the 1208 population covariance matrices Σ_{4h}, 1000 independent replications with sample size $N = 100, 300, 1000$ are drawn from $N_{11}(\mathbf{0}, \Sigma_{4h})$, respectively. The sample covariance matrix **S** from each replication is fitted by $\mathbf{M}_4$ of Example 4, and the sample F_{ML} is obtained by minimizing Equation (8) with Σ being replaced by **S**. For each of $8 \times 151 \times 3$ conditions of **c**, h and N, the average value of the sample F_{ML} across the 1000 replications is obtained. Thus, we have 151 values of average sample F_{ML} corresponding to the 151 values of h at each combinations of **c** and N.

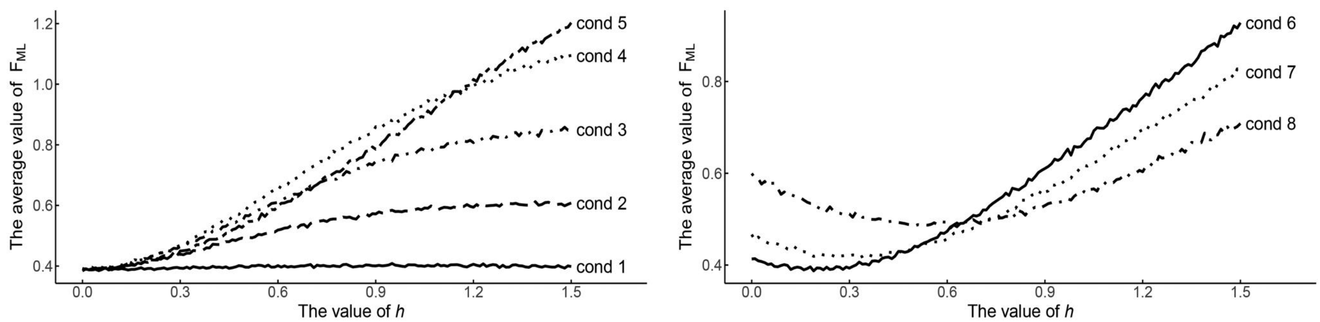
4.2. Results

Note that the numbers of excluded nonzero cross-loadings in conditions 1 to 5 are 5, 4, 3, 2 and 1, respectively. Figures 5–7



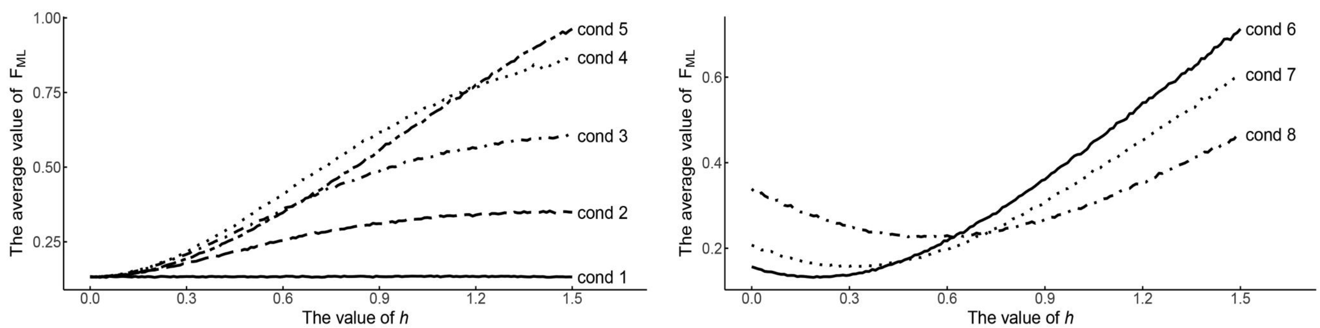
(a) Conditions 1 to 5 (b) Conditions 6 to 8

Figure 4. The changes of F_{ML0} as h varies in Example 4.



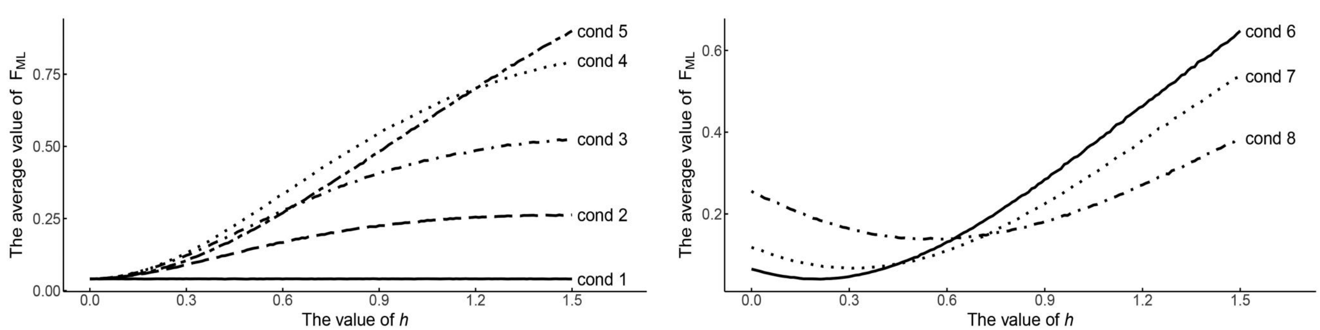
(a) Conditions 1 to 5 (b) Conditions 6 to 8

Figure 5. The changes of average value of sample F_{ML} as h varies, $N = 100$.



(a) Conditions 1 to 5 (b) Conditions 6 to 8

Figure 6. The changes of average value of sample F_{ML} as h varies, $N = 300$.



(a) Conditions 1 to 5 (b) Conditions 6 to 8

Figure 7. The changes of average value of sample F_{ML} as h varies, $N = 1000$.

show the changes of the average value of the sample F_{ML} as h varies for the 3 sample sizes. Overall, the average value of F_{ML} exhibits patterns consistent with their population counterpart in Figure 4. Specifically, Figures 5a–7a show that, conditional on h, empirical model-fit becomes better when a greater number of cross-loadings are excluded. Figures 5b–7b show that the average value of F_{ML} in each condition of **c** initially decreases and then increases with h.

Because smaller sample sizes are associated with larger sampling errors, the trajectories of the average value of F_{ML} at $N = 100$ in Figure 5 exhibit certain fluctuations, but become increasingly smooth as N increases. In particular, the curves become rather stable at $N = 300$ as shown in Figure 6, with little difference from their corresponding population counterparts in Figure 4. Thus, the patterns regarding the impact of excluding cross-loadings on model evaluation, as discussed at the population level in Section 3, also hold empirically.

5. Discussion and Conclusion

This article introduces a type of cross-loadings that are proportional to their target loadings, which can be accounted for by a rotation without changing the population covariance matrix of a unidimensional measurement model. This means that excluding such cross-loadings does not lead to model misspecification. However, the pattern changes when either the structural model is unsaturated or the cross-loadings are not proportional to their target loadings.

Both analytical and simulation results reveal that when the cross-loadings are not proportional to their target loadings, cross-loadings with greater number or larger size may be more likely to be absorbed by the structural model. This implies that, in practice, excluding a larger number or greater size of cross-loadings may lead to a better model-fit.

In SEM or factor analysis, it is often intuitively assumed that excluding more or larger cross-loadings results in more severe model misspecification, which in turn leads to poorer model-fit indices. Our findings indicate that this is not always the case. On the contrary, model-fit can sometimes improve as more or larger cross-loadings are excluded. We hope that this article provided a deeper theoretical insight into the impact of cross-loadings on overall model-fit.

The current study only uses the value of F_{ML} to assess model-fit. In practice, other fit indices such as RMSEA and CFI are also commonly employed (Bentler, 1990; Hu & Bentler, 1998, 1999; Steiger & Lind, 1980). In fact, once the population and model are specified, both the model degrees of freedom and the value of F_{ML} for the baseline model are determined. Therefore, a smaller F_{ML} corresponds to a smaller RMSEA and a larger CFI, and vice versa. As a consequence, the phenomenon identified in this study that excluding more or larger cross-loadings leads to better model-fit will also be reflected by other fit indices.

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