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Parameter Recovery for Misspecified Latent Mediation Models in the Bayesian Framework

Haiyan Liu, Ihnwhi Heo, Sarah Depaoli and Aleksandr Ivanov

University of California

ABSTRACT

Mediation analysis is widely used to examine whether a third variable mediates the relationship between two variables. Latent mediation analysis extends this framework to latent variables measured through observable indicators, combining measurement models and mediation paths. Accurate inference in latent mediation analysis depends on two critical assumptions: the correct specification of the measurement model and the absence of unmeasured confounders. However, these assumptions are often violated in practice. This study estimates latent mediation models within a Bayesian framework and investigates how specification errors and prior choices affect parameter estimates through a systematic simulation study. Specifically, we examine four typical specification errors: (1) misspecified (e.g., ignore cross-loading) measurement model for the mediator, (2) no measurement (e.g., using total score) models for the mediator, (3) no measurement models for confounders, and (4) ignoring confounders. We also evaluate the influence of prior specifications (diffuse vs. weakly informative) on Bayesian inference. The simulation results show that model misspecifications significantly affect the accuracy of mediation effect estimates. Standardized total scores for mediators and confounders attenuate mediation effects, bias parameter estimates, and produce inaccurate credible intervals. Ignoring confounders results in biased estimates, mainly when the confounding effects are medium- or large-sized. Adopting accurate weakly informative priors improves parameter recovery, coverage rates, and the ability to detect the true mediation effect compared to diffuse priors.

KEYWORDS

Bayesian estimation; latent mediation analysis; model misspecification; omitting confounders; prior specification

1. Introduction

Mediation analysis is a widely used tool for examining the causal mechanisms that link an independent variable to a dependent variable (Baron & Kenny, 1986; Hayes, 2009). Applications are spread across various fields, including epidemiology, psychology, sociology, and related disciplines (e.g., Fritz & MacKinnon, 2007; Liu et al., 2021). The primary objective of the mediation analysis is to determine whether the relationship between two variables X and Y is explained (or *mediated*), either wholly or partially, by a third variable (MacKinnon, 2012; Richiardi et al., 2013). For example, the relationship between maternal education and children's reading achievement is mediated by the home enrichment (Zadeh et al., 2010). In simple mediation analysis, where the variables are directly observed, regression-based methods are typically employed to investigate the relationships between the mediator, the independent variable, and the dependent variable.

Latent mediation analysis provides a robust framework for examining causal mechanisms among latent variables (Finch et al., 1997; Miočević et al., 2021). This approach is particularly beneficial in fields like psychology and the broader social sciences, where many important constructs, such as personality, intelligence, satisfaction, or stress, are not directly observable. Instead, these constructs are measured through various observable indicators, such as survey items or test scores (Anderson & Rubin, 1956; Cattell, 1952). Understanding the relationships among these constructs is of primary interest to researchers in these disciplines. Through latent mediation analysis, researchers can uncover the underlying mechanisms and causal pathways that drive observed behaviors and outcomes (Cai et al., 2023).

A latent mediation model has two fundamental components: the measurement and structural models (Finch et al., 1997). The measurement model defines the relationships between latent variables and their observed indicators, which is crucial to account for measurement error and to ensure that the constructs are accurately represented (Mulaik, 2009). This part of the model helps clarify how well the observed indicators reflect the underlying latent constructs. The structural model then builds on this foundation by capturing the direct and indirect effects among the latent variables. This aspect of the model allows researchers to untangle complex causal pathways, providing a detailed understanding of how latent variables influence each other (Derkach et al., 2019). By mapping out these pathways, the structural model enables researchers to gain deeper insight

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into the underlying mechanisms that drive observed relationships, revealing how different latent constructs interact to produce specific outcomes.

The Bayesian approach provides a flexible framework for the parameter estimation and evaluation in mediation analysis (Liu et al., 2021; Wang & Preacher, 2015; Yuan & MacKinnon, 2009). Since Yuan and MacKinnon (2009) introduced the Bayesian framework to mediation analysis, various methodological advancements have been made within this perspective. Enders et al. (2013) evaluated the performance of Bayesian estimation in handling mediation effects when data are missing, demonstrating its robustness under such conditions. Liu et al. (2023) proposed a general framework for Bayesian hypothesis testing of mediation effects using Bayes factors and investigates the potential impact of prior odds specifications on Bayesian hypothesis. Laghaie and Otter (2023) suggested employing Bayes factors as a measure of conditional independence between treatment and outcome to strengthen the causal mediation inference. Additionally, Daniels et al. (2012) applied a nonparametric Bayesian approach to examine causal mediation effects, highlighting its potential for modeling complex relationships. Collectively, these studies demonstrate the adaptability and effectiveness of the Bayesian framework in addressing a range of challenges in mediation analysis.

Bayesian estimation involves specifying prior distributions for model parameters and updating these priors with observed data through Bayes' theorem to obtain posterior distributions (Gelman et al., 2014). This incorporation of priors can address issues of insufficient information, particularly in scenarios involving complex models or small sample sizes, where convergence might otherwise be challenging to achieve (Depaoli et al., 2019; Liu et al., 2022). The impact of priors on inferences depends on their accuracy and informativeness. An accurate prior is centered near the true parameter value, whereas an inaccurate prior deviates significantly, potentially introducing bias into the posterior estimates. Informative priors have a smaller variance, reflecting greater certainty about the parameter value, while diffuse priors have a larger variance, indicating less certainty and providing minimal guidance to the estimation process. The accuracy and informativeness of priors may impact the validity of inference in latent mediation analysis. For instance, Miočević et al. (2021) examined how the accuracy of priors for structural paths and factor loadings influences point and interval estimates of the mediation effect in a single-mediator latent mediation model without confounders. Furthermore, Miočević and Golchi (2022) proposed an objective procedure for creating informative priors for mediation analysis based on historical data, improving precision and power to detect mediation effects.

The accurate estimation of indirect or mediation effects in Bayesian latent variable mediation analysis relies on the satisfaction of several key assumptions. First, there should be no unmeasured confounders for the paths between the latent independent, mediator, and dependent variables. All confounders should be measured and controlled in the analysis. Unmeasured confounders can introduce bias in the estimates of path coefficients, distorting the actual mediation effect. Second, the measurement model of the latent variable should be correctly specified. Misspecification in any part of the measurement model can lead to an incorrect understanding of the latent factors and bias the inference of the mediation effect. These assumptions are often violated in practice. Unmeasured confounders are common in observational studies, where not all relevant variables can be controlled or measured. In addition, many variables are measured with error due to various reasons, such as imperfect instruments and incorrect theoretical structures of the measurement model.

The violation of model assumptions has led to investigations into the impact of omitted confounders and measurement errors in the mediator, particularly in simple mediation analysis. Measurement error arises when the mediator is not perfectly reliable. Fritz et al. (2016) investigated the impact of the measurement error and omitted confounders on mediation effect estimates, showing that these factors can lead to overestimation, underestimation, or, in some cases, unbiased estimation within the frequentist framework. Similarly, Liu and Wang (2021) examined the effects of measurement error and omitted confounders on statistical inference of mediation effects, proposing a sensitivity analysis procedure to mitigate these issues. Lastly, Zhang and Wang (2024) argue that when the confounders of the mediator M and the dependent variable Y are not taken into account in the mediation analysis, their residuals become correlated. To address this issue, they propose using informative priors with a mean of 0 and a small variance (e.g., 0.01) for the correlation parameter between the residuals. Taking into account this correlation, the analysis can partially mitigate the bias introduced by omitted confounders, offering a practical solution when direct measurement of confounders is not possible.

Incorporating latent variables into the mediation path adds complexity, requiring correct specification of their measurement models. Measurement models can be misspecified due to incorrect theoretical assumptions, with typical errors, including ignoring cross-loadings, and ignoring the measurement model by using standardized total scores rather than explicitly modeling the measurement structure of the latent variable. The standardized total scores are created by adding the observed indicators and rescaling the total score (mean = 0, standard deviation = 1). Ignoring a cross-loading is a type of misspecification often arises in applied research when theoretical assumptions oversimplify the relationships between observed indicators and latent variables. The standardized total score condition reflects a common practice in applied research, where composite scores are used in place of latent variables due to sample size constraints, model convergence challenges, or software limitations. While latent variable modeling is ideal, composite scores introduce measurement error by ignoring indicator covariance, potentially distorting mediation estimates (Bauer & Curran, 2016; McNeish & Wolf, 2020). Researchers may also resort to composite scores when a single-factor model does not fit well. Furthermore, most existing literature focuses on the measurement error in the mediator and dependent variable. However, few studies have investigated measurement errors in confounders, which may also impact the inference of the mediation effect.

Despite their critical importance, the impact of model assumption violations has not yet been fully explored in latent mediation analysis. It remains to be seen how the misspecification of the measurement model in different parts of the mediation model impacts the inference of the mediation effects. Comprehensive evaluations of the effects of measurement model misspecification and the omission or presence of latent confounders on the inference of mediation effects in latent mediation analysis are still needed. Moreover, the presence of a confounder, of which the measurement model could also be misspecified, further complicates the inference of the mediation effect. In the Bayesian context, the accuracy and informativeness of priors may interact with model misspecification, additionally affecting the estimates of mediation effects. For example, omitting a confounder in the mediator–to–outcome path ($\eta_M \rightarrow \eta_Y$) can inflate the estimated path coefficient. If a prior is centered away from the true value, it may exacerbate the bias caused by the omission of confounders. Conversely, a weakly informative prior with a reasonable center can help mitigate some of this bias. This emphasizes the importance of careful prior selection, particularly in cases where model assumptions may be violated.

To address the current gap in the literature, this study conducts a simulation study to systematically assess the impact of misspecification in measurement models of latent variables and the omission of latent confounders on the valid inference of mediation effects within the framework of latent mediation analysis. Our unique focus on the measurement models of mediators and latent confounders and the presence of unmeasured latent confounders will shed new light on the accurate inference of the mediation effect. Additionally, we will investigate the performance of Bayesian estimation methods under various prior specifications to determine how different priors affect parameter estimates and overall model performance. The findings of the simulation study will provide valuable insights into the robustness of latent mediation analysis and offer guidelines for improving the accuracy of mediation effect estimation using latent variables.

The remainder of this article is organized as follows: First, we overview the simple and latent mediation models. Next, we delve into Bayesian estimation methods within a general framework. We then outline our simulation design and describe various types of model misspecification. Following this, we present the simulation results. Finally, we conclude the study by discussing current developments relevant to the application of these models, as well as future methodological research directions.

2. Latent Mediation Analysis

This section will begin with a brief overview of simple mediation analysis and its extension to latent mediation analysis. We will also discuss the model assumptions and potential misspecifications in latent mediation analysis.

2.1. Simple Mediation Analysis

We briefly introduce the simple mediation analysis in a linear regression context. For notation, we use the three-variable system in which an independent variable X predicts a dependent variable Y via regression models (Baron & Kenny, 1986), and a mediator M is included; this is demonstrated by the diagrams in Figure 1. The diagram on the top panel of Figure 1 portrays the total relation between the independent and dependent variables, and the regression equation is as follows:

$$\text{Model 1:} \quad Y_i = i_1 + cX_i + \varepsilon_{i,1}, \tag{1}$$

where the coefficient c is the *total effect* of the independent variable X on the dependent variable Y (not considering M), i_1 is the intercept of the model and $\varepsilon_{i,1}$ is the error term for each i. The bottom panel of Figure 1 is a mediation model with the variable M as mediator. To study the indirect effect of X on Y through a mediator variable M, one needs to regress M on X and then Y on both X and M,

$$\text{Model 2:} \quad M_i = i_2 + aX_i + \varepsilon_{i,2} \tag{2}$$

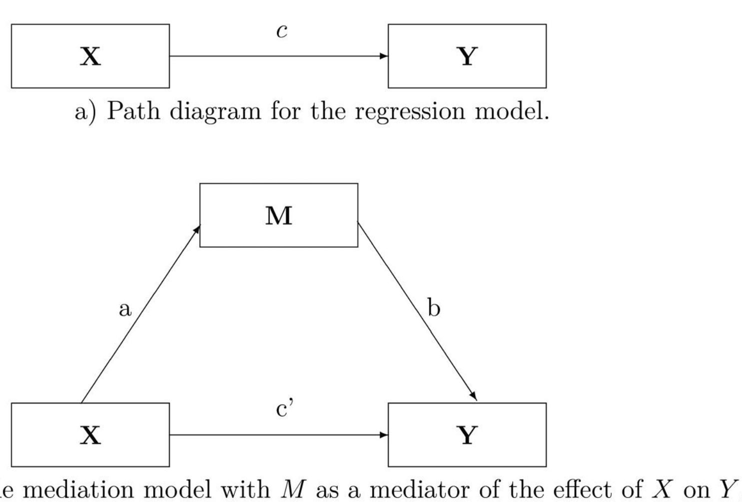

Figure 1. Path diagrams for the regression model and the mediation model.

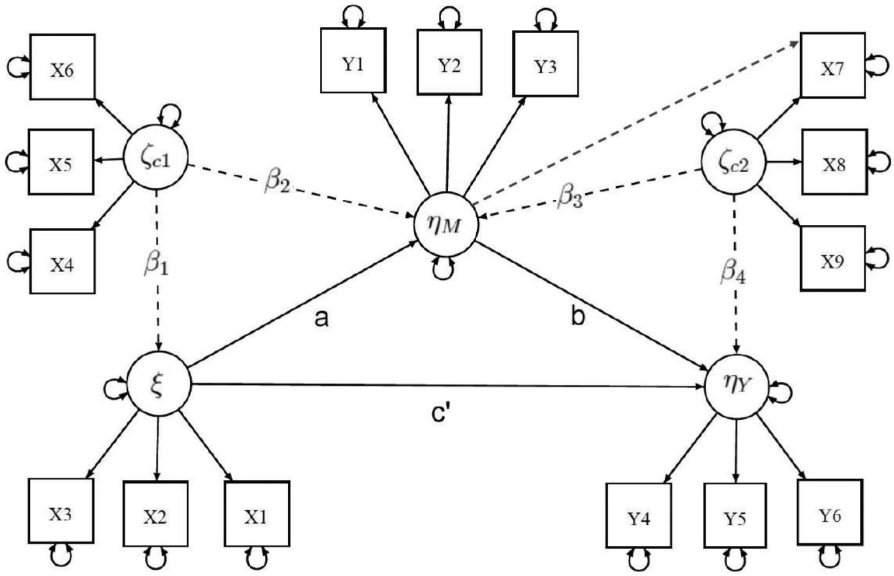

Figure 2. Latent mediation model with three indicators per latent variable. The dashed lines indicate either the potential cross-loading or the paths of the potential confounders.

$$\text{Model 3:} \quad Y_i = i_3 + bM_i + c'X_i + \varepsilon_{i,3}, \tag{3}$$

where i_2 and i_3 are the intercepts of the two regression models. The parameter a is the coefficient of the relation between X and M, b is the coefficient that relates the mediator M to Y while controlling X, and c' is the coefficient quantifying the relationship between X and Y while controlling M. The two terms $\varepsilon_{i,2}$ and $\varepsilon_{i,3}$ are errors associated with case i in these two models.

The *indirect effect* is the estimate of the reduction in the predictor effect on the outcome variable when the mediator is included in the model, that is, $\hat{c} - \hat{c}'$ given a sample. In general, it holds that $\hat{c} - \hat{c}' = \hat{a} \times \hat{b}$ when the three variables are linearly related to each other (MacKinnon et al., 1995). The rationale behind this method is that the mediation effect depends on the degree to which the predictor changes the mediator, represented by the coefficient a, and the extent to which the mediator affects the outcome variable, represented by the coefficient b.

To fully form the mediation model, the path from X to M to Y should be causal. Specifically, there should be no unmeasured confounders for the X to Y relationship, the X to M relationship, and the M to Y relationship (Imai et al., 2010; VanderWeele, 2015; VanderWeele & Vansteelandt, 2009). In addition, if there are confounders for M and Y, they should not be affected by the independent variable X.

2.2. Latent Mediation Analysis with Latent Confounders

In the realm of social and psychological sciences, researchers often focus on latent traits and their relations. Psychological traits are typically assessed using measurement scales, allowing the measurement error to be addressed. The mediation model is naturally expanded to incorporate latent variables in this context. Our illustration of latent mediation models is based on the single mediator model with latent variables, as introduced by Finch et al. (1997) and Miočević et al. (2021). The latent mediation model consists of a measurement model for the independent variable, the mediator, and the outcome variable, along with a structural model for the indirect and direct effects among them. In the current study, we also consider the presence of potential latent confounders for the paths from the independent variables to the mediator and from the mediator to the outcome variable, respectively.

We present Figure 2 for an example of the latent mediation model with three indicators per latent variable.

In this latent mediation model, we have ξ as the latent independent variable, η_M as the latent mediator, and η_Y as the latent outcome variable. To ensure generalizability, we also consider two latent confounders: ζ_{c1} for the path between ξ and η_M, and ζ_{c2} for the path between η_M and η_Y.

We describe the measurement model for the latent independent variable ξ[1], the two latent confounders ζ_1 and ζ_2, and the measurement model for the mediator η_M[2], and the latent dependent variable η_Y in the following:

$$\begin{bmatrix} x_1 \\ \vdots \\ x_9 \end{bmatrix} = \Lambda_x \begin{bmatrix} \xi \\ \zeta_1 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_9 \end{bmatrix}, \begin{bmatrix} y_1 \\ \vdots \\ y_6 \end{bmatrix} = \Lambda_y \begin{bmatrix} \eta_M \\ \eta_Y \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_6 \end{bmatrix} \tag{4}$$

The measurement model for the independent variable ξ and the two confounders includes a 9×3 factor loading matrix Λ_x. The errors δ_i $(i = 1, \ldots, 9)$ are assumed to follow independent normal distributions, denoted as $N(0, \sigma^2_{\delta_i})$. For mediator η_M and dependent variable η_Y, each has 3 indicators, characterized by a 6×2 factor loading matrix Λ_y. The corresponding error terms ε_j $(j = 1, \ldots, 6)$ are also follow independent normal distributions, $N(0, \sigma^2_{\varepsilon_j})$.

[1]The independent variable ξ could be either exogenous or endogenous, depending on the presence of a non-zero path from ζ_{c1} to ξ.

[2]The formulation assumes the absence of cross-loading; however, it can be generalized to include cross-loadings.

The mediation path among the latent variables is represented by the following notation:

$$\begin{bmatrix} \xi \\ \eta_M \\ \eta_Y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ c' & b & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \eta_M \\ \eta_Y \end{bmatrix} + \begin{bmatrix} \beta_1 & 0 \\ \beta_2 & \beta_3 \\ 0 & \beta_4 \end{bmatrix} \begin{bmatrix} \zeta_{c1} \\ \zeta_{c2} \end{bmatrix} + \begin{bmatrix} e_\xi \\ e_M \\ e_Y \end{bmatrix} \quad (5)$$

The independent variable ξ is linked to the mediator η_M through a. The symbols b and c' are the coefficient from the mediator η_M and the independent variable ξ to the dependent variable η_Y. The residuals or errors of the paths are symbolized as e_ξ, e_M and e_Y, and they follow an independent normal distribution with mean 0 and variance parameters σ^2_ξ, σ^2_M, and σ^2_Y,

$$\begin{bmatrix} e_\xi \\ e_M \\ e_Y \end{bmatrix} \sim N_3 \left(\mathbf{0}, \begin{bmatrix} \sigma^2_\xi & 0 & 0 \\ 0 & \sigma^2_M & 0 \\ 0 & 0 & \sigma^2_Y \end{bmatrix} \right). \quad (6)$$

In the latent mediation model, the mediation (or indirect) effect is defined as $a \cdot b$. This indirect effect (i.e., $a \cdot b$) represents the pathway through which the independent variable ξ influences the dependent variable η_Y via the mediator η_M. It quantifies how changes in the independent variable affect the mediator, which in turn affects the dependent variable. The direct effect c' measures the influence of the independent variable on the dependent variable that is not mediated by η_M. When fitting the latent mediation model to an empirical dataset, the goal is to estimate both the indirect effect $a \cdot b$ and the direct effect c', providing insights into the underlying mechanisms of the relationship between the independent and dependent variables.

2.3. Model Assumptions

Both simple and latent mediation models are essential tools for estimating and testing the indirect or mediation effect, denoted as $\widehat{ab}$. However, it is crucial to recognize that the observed effect may not inherently represent the true mediation effect. As discussed by VanderWeele and Vansteelandt (2009) and VanderWeele (2015), several vital assumptions must be satisfied to ensure valid inference.

First, it is essential that there must be no unmeasured confounders affecting the relationships between the independent variable (X) and the mediator (M), the mediator (M) and the outcome variable (Y), and the independent variable (X) and the outcome variable (Y). These assumptions are critical because the presence of unmeasured confounders can introduce bias, making the accurate estimation of the mediation effect difficult.

In addition, the latent mediation model introduces additional complexity due to the incorporation of latent variables. For the latent mediation model, it is crucial to ensure the measurement models are correctly specified. Any misspecification in the measurement models can lead to a biased understanding of the latent variables, which in turn can bias the estimates of the mediation effect.

Meeting these assumptions can be challenging in practice. Identifying and controlling for unmeasured confounders is often difficult, and accurately specifying the measurement model requires careful consideration and validation. Despite these challenges, it is important to adhere to these assumptions to obtain valid and reliable estimates of mediation effects in both simple and latent mediation models. When these model assumptions are violated, the model becomes misspecified, resulting in a poor fit to the empirical data. Thus, violations of model assumptions fundamentally reflect issues of model specification.

3. Bayesian Estimation

The Bayesian estimation framework provides enhanced flexibility for estimating SEM models, as shown by Muthén and Asparouhov (2012). It is particularly advantageous for complex models, where traditional estimation methods may face challenges, such as convergence issues or limited sample sizes (Depaoli, 2013). These benefits have positioned Bayesian estimation as a valuable and increasingly utilized approach within SEM.

Latent mediation models include both measurement models and the structural model describing the relationship among latent variables. Thus, they can be fitted as Bayesian SEM models. Miočević et al. (2021) evaluated the performance of Bayesian inference for latent mediation models under correct model specifications.

Bayesian estimation incorporates prior beliefs for each parameter and updates the "belief" with the collected data through the Bayesian theorem:

$$P(\boldsymbol{\theta}|\text{data}) \propto P(\text{data}|\boldsymbol{\theta})P(\boldsymbol{\theta}) \quad (7)$$

where $\boldsymbol{\theta}$ is the collection of all model parameters to be estimated, $P(\boldsymbol{\theta}|\text{data})$ is the posterior distribution, $P(\text{data}|\boldsymbol{\theta})$ is the likelihood, and $P(\boldsymbol{\theta})$ is the prior of the model parameters.

3.1. Prior Specification

In a latent mediation model, model parameters include: the factor loadings λ_x's and λ_y's, path coefficient among latent factors $a, b, c', \beta_1, \beta_2, \beta_3$ and β_4, residual variances of latent factors $\sigma^2_{\xi/\eta_M/\eta_Y}$, and residual variances of the indicators $\sigma^2_{\varepsilon_j}$'s.

In Bayesian estimation, the choice of priors plays a crucial role in guiding parameter estimation and ensuring stable model performance, particularly in complex models like latent mediation analysis. In our study, we selected normal priors for the factor loadings and path coefficients as commonly used in both SEM and mediation analysis (e.g., Depaoli, 2013; Yuan & MacKinnon, 2009),

$$\lambda_{x/y} \sim \mathcal{N}\left[\mu_{\lambda_{x/y}}, \sigma^2_{\lambda_{x/y}}\right], \quad (8)$$

$$a, b, c', \beta_k \sim \mathcal{N}\left[\mu_{\lambda_{a/b/c'/\beta_k}}, \sigma^2_{a/b/c'/\beta_j}\right] \quad (9)$$

The normal distribution, being symmetric, is well-suited for representing prior knowledge or uncertainty about continuous parameters. The accuracy of a normal prior is determined by the mean hyperparameter (μ), which, when set to the true parameter value, results in an accurate prior. Conversely, an inaccurate prior arises when μ deviates from the true value, with the degree of inaccuracy increasing as the deviation grows. It is worth noting that in practice, researchers do not know the true parameter value, so specifying an "accurate" prior requires setting μ based on plausible parameter values informed by prior knowledge, theory, or previous studies. In addition, the variance hyperparameter (σ^2) controls the informativeness of the prior by determining its spread. A larger variance (e.g., $\sigma^2 = 10^4$) represents a diffuse prior, which provides minimal guidance and allows the data to primarily influence the estimation process. In contrast, a smaller variance (e.g., $\sigma^2 = 0.01$) results in a weakly informative prior that incorporates moderate prior certainty without being overly restrictive, thereby balancing prior knowledge and data-driven inference.

For the residual variance parameters ($\sigma^2_{\xi/\eta_M/\eta_Y/\varepsilon_j}$), we specify the inverse gamma ($\mathcal{IG}$) prior,

$$\sigma^2_{\xi/\eta_M/\eta_Y/\varepsilon_j} \sim \mathcal{IG}(a, b), \tag{10}$$

The inverse gamma distribution is defined over positive values, making it suitable for variance parameters, which must always be non-negative. The shape (a) and scale (b) hyper-parameters control the informativeness of the $\mathcal{IG}$ prior. Small values of a and b (e.g., $a = b = 0.01$) produce weakly informative priors, ensuring minimal prior influence when little is known about the variances. Larger values, however, reflect greater certainty about the plausible range of variance estimates. This flexibility allows the $\mathcal{IG}$ prior to accommodate varying levels of prior knowledge while ensuring computational stability (e.g., Depaoli et al., 2024; Liu et al., 2016).

3.2. Posterior Inference

For SEM models, the posterior distribution often lacks a closed-form solution due to the complexity of the model structure and parameter dependencies. Therefore, Markov Chain Monte Carlo (MCMC) methods are frequently employed to approximate the posterior distribution by generating samples from it. MCMC techniques, such as Gibbs sampling and the Metropolis-Hastings algorithm, iteratively draw samples that converge to the target posterior distribution (Casella & George, 1992; Hastings, 1970). These posterior samples are then used to estimate parameter values, credible intervals, and other relevant quantities of interest.

In the simulation study, we will evaluate the robustness of parameter recovery by examining the effects of different hyperparameter settings for the normal priors. Specifically, we will vary the priors' accuracy (centered near or far from the true value) and informativeness (tight versus diffuse spread) to assess their influence on posterior estimates. This allows us to systematically investigate the sensitivity of Bayesian SEM estimation to prior specifications.

Table 1. Summary of simulation factors and prior specifications.

Simulation Factor	Levels/Specifications
Mediation Effect	$a = b = 0.14$ (small)
	$a = b = 0.39$ (medium)
	$a = b = 0.59$ (large)
Confounding Effect	$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ (none)
	$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.14$ (small)
	$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.39$ (medium)
Sample Size	$n = 100, 200, 400$
Model Specification	**True Model**
	Misspecified Models:
	- Wrong measurement model for mediator factor
	- No measurement model for mediator factor
	- No measurement model for one confounder
	- No measurement model for two confounders
	- One ignored confounder factor
	- Two ignored confounder factors
Prior Specification	**Diffuse Prior:**
	- $\mathcal{IG}(-1, 0)$ for residual variances
	Weakly Informative Priors:
	- Accurate: $\mathcal{N}(\text{true}, 0.5\text{true})$
	- Inaccurate-1SD: $\mathcal{N}\left(\text{true} + \sqrt{0.1\text{true}}, 0.5\text{true}\right)$
	- Inaccurate-2SD: $\mathcal{N}\left(\text{true} + 2\sqrt{0.1\text{true}}, 0.5\text{true}\right)$

4. Simulation Design

In this section, we will describe the simulation study, which was used to assess the impact of model assumption violations on parameter estimates within the context of Bayesian latent mediation analysis. Specifically, we will investigate the effects of measurement model misspecification in the mediator or the confounder, the omission of confounders, and how these interact with the accuracy and informativeness of prior distributions. In the following sections, we describe the population model used for data generation, the parameter levels considered, the sample sizes, the manipulation of model misspecifications, and the priors employed in the simulation study. A summary of all simulation conditions is provided in Table 1.

4.1. Population Model

We specified a single population model with confounders, described in Figure 2. The model includes a latent independent variable (ξ), a dependent variable (η_Y), and a latent mediator (η_M). Each latent variable is measured by three primary continuous items: ξ defined by Items X_1 to X_3, η_M by Items Y_1 to Y_3, and η_Y by Items Y_4 to Y_6. Paths are defined from ξ to η_M and η_M to η_Y with respective coefficients a and b respectively, and a direct path from ξ to η_Y with coefficient c'.

To examine the impact of confounder omission, we include two confounders, ζ_{c1} and ζ_{c2}, each with three primary indicators: ζ_{c1} loads on Items X_4 to X_6, and ζ_{c2} on Items X_7 to X_9. ζ_{c1} confounds the paths from ξ to η_M with coefficients β_1 and β_2, and ζ_{c2} confounds the paths from η_M to η_Y with coefficients β_3 and β_4.

The primary factor loadings of all items are set as 0.7. We also included a non-zero cross-loading from Item X_7 to η_M with a value of 0.5. The factor variance for ζ_{c1} and ζ_{c2} is set at 1. The variance or residual variance of ξ is set at $1 - \beta_1^2$, resulting in values of 1, 0.9804, or 0.8479 depending on the value of β_1, to ensure that the variance of ξ is equal to 1.

The variances of η_M and η_Y account for both the variance explained by their predictor factors and residual variance. For η_M, the residual variance is calculated as $1 - a^2 - \beta_2^2 - \beta_3^2$, while for η_Y, it is $1 - b^2 - c'^2 - \beta_4^2$. These calculations make sure that the total variances for η_M and η_Y are equal to 1. The residual variance of each indicator is set to 0.51, resulting in a unit variance for every indicator.

In this setup, the population parameters were specified in a standardized manner, ensuring that the total variance of each indicator and latent factor equals 1. As a result, the estimated mediation effect represents a standardized effect, facilitating meaningful interpretation and comparison across different conditions.

4.2. Design Factors and Prior

The simulation design factors we consider involve manipulating the mediation effect, confounding effect, sample size, types of misspecification, and prior specifications.

4.2.1. Mediation Effect

In the population model, the coefficient c' is fixed at 0.14. The coefficients a and b are assigned values of 0.14, 0.39, and 0.59. These values correspond to mediation effects $a \cdot b$ of 0.01496, 0.1521, and 0.3481, representing small, medium, and large effects, respectively (Liu et al., 2021).

4.2.2. Confounding Effect

We denote the path coefficients from the confounders ζ_{c1} and ζ_{c2} to ξ and η_M, as well as η_M and η_Y as β_1, β_2, β_3, and β_4. These coefficients take values of 0, 0.14, and 0.39, which represent no confounding effect, a minor confounding effect, and a medium confounding effect, respectively. For simplicity, we set $\beta_1 = \beta_2 = \beta_3 = \beta_4$.

4.2.3. Sample Size

Three sample sizes ($n = 100$, 200, and 400) are considered to reflect the range typically found in both methodological and applied research (e.g., Miočević et al., 2021).

4.2.4. Model Specification

The path diagram of the population model is provided in Figure 2. Each set of parameter values represents a "true model" for the respective simulation conditions.

In our simulation design, we examine six misspecified models to cover different scenarios of model misspecifications across measurement and structural models. Firstly, we consider two types of measurement model misspecifications for the mediator factor η_M: one that omits the cross-loading from η_M to Item X_7 (referred to as the "wrong measurement model for mediator factor") and another that uses the standardized total score of the indicators for η_M (referred to as the "no measurement model for mediator factor"). Omitting the cross-loading from η_M to Item X_7 enables us to evaluate the impact of disregarding the shared variance between a confounder and the mediator on parameter estimates.

Ignoring cross-loadings is a common form of model misspecification in structural equation modeling and has been extensively examined in prior research as a prevalent issue that can lead to biased parameter estimates and misinterpretation of latent constructs (e.g., Cain & Zhang, 2019; Depaoli et al., 2024; Winter & Depaoli, 2022). This type of misspecification often arises in applied research when theoretical assumptions oversimplify the relationships between observed indicators and latent variables, potentially leading to inaccurate conclusions. Similarly, the practice of using standardized total scores as proxies for latent variables is frequently observed in empirical studies due to its simplicity and ease of implementation, despite its tendency to overlook measurement error and the true underlying structure of the construct (Bauer & Curran, 2016; McNeish & Wolf, 2020).

In addition to the misspecification of the mediator factor, we consider the misspecification of the measurement models of the confounder factors: models that ignore the measurement structure of either ζ_{c1} (referred to as "no measurement model for one confounder") or both ζ_{c1} and ζ_{c2} (referred to as "no measurement model for two confounders"). In these "no measurement model" conditions for the confounders, standardized total scores are used as proxies for the latent confounders.

For structural model misspecifications, we consider models that ignore the presence of a confounder for the path from ξ to η_M (referred to as "one ignored confounder factor") and models that omit both confounders ζ_{c1} and ζ_{c2} (referred to as "two ignored confounder factors"). In practical applications, measuring all relevant confounders is almost impossible. Hence, it is necessary to manipulate it in the simulation.

4.2.5. Prior Specification

For the variance parameter, the inverse gamma (IG) prior is a widely used choice in Bayesian structural equation modeling. As demonstrated by Asparouhov and Muthén (2010), the priors $\mathcal{IG}(-1, 0)$, $\mathcal{IG}(0, 0)$, and $\mathcal{IG}(1, 2)$ have minimal influence on parameter estimates, especially when the model includes a reasonable number of indicators (e.g., five indicators per factor). Notably, the $\mathcal{IG}(-1, 0)$ prior is equivalent to a uniform prior on variance parameters, rendering it non-informative. This makes it the default choice in M*plus*, as it allows the data to primarily inform the parameter estimates without imposing strong prior assumptions. Given these considerations, we adopted the $\mathcal{IG}(-1, 0)$ prior in our simulation study to align with established best practices and to reduce the potential for prior-induced biases.

For factor loadings and path coefficients, we selected normal priors based on previous studies such as Asparouhov and Muthén (2010) and Depaoli (2013), which demonstrated that both the accuracy and informativeness of normal priors can influence estimation outcomes. When normal priors are highly diffuse, such as $\mathcal{N}(0, \infty)$ (the default prior in M*plus*), they exert minimal influence on point estimates, even when their center deviates from the true value. However, a certain level of informativeness is necessary to provide meaningful guidance in estimation, particularly in small-sample or complex models.

Following the framework proposed by Depaoli (2013), we consider three weakly informative priors,

$$\text{Weakly informative-accurate}: \lambda_{x/y}, \beta, a, b, c' \sim \mathcal{N}(\text{true}, 0.5\text{true}) \tag{11}$$

$$\text{Weakly informative-inaccurate-1SD}: \lambda_{x/y}, \beta, a, b, c' \sim \mathcal{N}\left(\text{true} + \sqrt{0.1\text{true}}, 0.5\text{true}\right) \tag{12}$$

$$\text{Weakly informative-inaccurate-2SD}: \lambda_{x/y}, \beta, a, b, c' \sim \mathcal{N}\left(\text{true} + 2\sqrt{0.1\text{true}}, 0.5\text{true}\right) \tag{13}$$

These three weakly informative priors share the same level of informativeness, with the prior variance set at 50% of the true parameter value. The priors differ in terms of accuracy, with their centers positioned at the true value (accurate), $\sqrt{0.1 \times \text{true}}$ (mildly inaccurate), and $2\sqrt{0.1 \times \text{true}}$ (more inaccurate). This design allows us to systematically assess how varying degrees of prior accuracy influence parameter recovery, coverage rates, and model performance, offering practical insights into the trade-offs between prior informativeness and bias in Bayesian latent mediation analysis.

4.3. Summary

Table 1 summarizes all the simulation factors and prior specifications. We consider three levels for mediation paths, three levels for confounding effects, three sample sizes, seven model specifications, and four prior specifications, resulting in $3 \times 3 \times 3 \times 7 \times 4 = 756$ simulation cells. For each cell, 500 datasets were generated, and true and misspecified models were fitted to all replicated datasets. The simulation was conducted in M*plus* (Muthén & Muthén, 1998–2017). The percentage of converged replications was around 99.67%, with a median of 100% based on the Gelman-Rubin ($\hat{R}$) convergence statistic. The simulation results will be calculated based on the converged replications.

In the following section, we will assess the accuracy of parameter estimates, the 95% credible interval coverage rates, the proportion of replications where the credible interval excludes zero, and variations in the root mean square error (RMSE).

5. Simulation Results

In this section, we will report on how model misspecifications in different parts of the model and the omission of latent confounders impact the Bayesian inference of the mediation effect. We will provide detailed information on the accuracy of point estimates, a summary of the credible intervals, and the power of Bayesian methods to detect the mediation effect. Additionally, we will evaluate how the performance of the Bayesian estimation method varies under different prior specifications.

5.1. Evaluation Criteria

Posterior inference is based on samples drawn from the posterior distribution. Two Markov chains are generated for each parameter. After the burn-in phase of 10,000 iterations, each chain consisted of 10,000 iterations for the estimated posterior. The summary statistics are computed based on a total of 20,000 iterations (10,000 from each chain).

The posterior mean based on the samples is computed as

$$\hat{\theta} = \frac{1}{20000}\sum_{i=1}^{20000} \theta^{(i)}. \tag{14}$$

Given a significance level α, a posterior credible interval of rth replication is defined as interval $[L_r, R_r]$ such that

$$\frac{\#\{\theta^{(i)} : \theta^{(i)} < L_r\}}{20000} = \frac{\#\{\theta^{(i)} : \theta^{(i)} > R_r\}}{10000} = \alpha/2. \tag{15}$$

5.1.1. Relative Bias

Let θ be an arbitrary parameter in the model to be estimated and also its population value. Let $\hat{\theta}_r$ and $[L_r, R_r]$ be the posterior mean and 95% credible interval from the rth replication. Let R be the number of replications that converged[3], then

$$\bar{\theta} = \frac{1}{R}\sum_{r=1}^{R} \hat{\theta}_r, \tag{16}$$

which is the average of parameter estimates across R converged replications.

The accuracy of parameter estimates is evaluated using "relative bias," which is a ratio of bias (difference between a point estimate and the true value of a parameter) to the absolute true value,

$$\text{relative bias}_\theta = \begin{cases} \dfrac{\bar{\theta} - \theta}{|\theta|} & \text{if } \theta \neq 0 \\ (\bar{\theta} - \theta) & \text{otherwise.} \end{cases} \tag{17}$$

5.1.2. Coverage Rates of Bayesian Credible Interval

Coverage rates refer to the proportion of times that the true parameter value is captured within the estimated credible interval in repeated sampling or replications. It is a crucial metric in Bayesian analysis for assessing the reliability and validity of interval estimates. In the current study, we will report the coverage rate of the 95% credible interval.

Let θ be a parameter and its true value. The coverage rate (CR) for θ is defined as:

$$\text{CR}_\theta = \frac{1}{R}\sum_{r=1}^{R} I(\theta \in [L_r, R_r]) \tag{18}$$

where $[L_r, R_r]$ is the 95% Bayesian credible interval, R is the number of converged replications, and $I(\cdot)$ is an indicator function that takes a value of 1 if the checking condition is

[3]The convergence is assessed based on the Proportional Scale Reduction (PSR) factor less than 1.1 for all parameters in the model.

true. The coverage rate is often used to assess the validity of the Bayesian credible intervals. A coverage rate close to the nominal one (i.e., 0.95) indicates that the statistical inference based on credible intervals is trustworthy.

5.1.3. Bayesian "Power" for Detecting Mediation Effect

To evaluate the ability of Bayesian estimation methods to detect the mediation effect, we calculated the proportion of replications in which the 95% credible interval excludes zero. This index quantifies the posterior probability of detecting the mediation effect and is comparable to the "power" of a study in the frequentist framework. While the term "power" is more traditionally associated with the frequentist framework, we use it here to facilitate comparisons and enhance interpretability for readers familiar with frequentist terminology.

$$\text{power} = \frac{1}{R}\sum_{r=1}^{R} I(0 \notin [L_r, R_r]) \tag{19}$$

5.1.4. Root Mean Square Error

The RMSE is a metric used to quantify the average difference between estimated values and true values. Let $\hat{\theta}_r$ be the posterior mean of parameter θ (also its true value is also denoted as θ) in the r'th replication, then

$$\text{RMSE} = \sqrt{\frac{1}{R}(\hat{\theta}_r - \theta)^2} \tag{20}$$

RMSE reflects how well the central tendency (posterior mean) of the posterior distribution aligns with the true value and the consistency of these estimates across all replications. Lower RMSE values indicate more accurate and reliable estimates, whereas higher RMSE values suggest greater discrepancies between the estimated and actual values.

In the following sections, we present results on the accuracy of parameter estimates, coverage rates of credible intervals, power to detect the mediation effect, RMSE, and prior sensitivity analysis. Within each category, we examine results for the following conditions: (1) the correctly specified model, (2) models with a misspecified or absent measurement model for the mediator, (3) models without a measurement model for the confounders, and (4) models that omit confounders. Notably, we report results for the misspecified measurement model and the no-measurement model for the mediator in the same subsection, as both conditions introduce measurement errors in the mediator.

5.2. Accuracy of Parameter Estimates

Figure 3 displays the relative bias in the estimates of the mediation/indirect effect with diffuse priors across seven models, as outlined in Table 1. The two black horizontal lines are at −0.1 and 0.1, representing a 10% bias below and above the true value. A relative bias within this range is considered acceptable.

The columns of the grid represent different levels of the mediation effect ($a \cdot b = 0.0196$, 0.1521, and 0.3481), corresponding to small, medium, and large mediation effects. The rows reflect the magnitude of the paths of the confounders, which range from 0 (no confounding) to 0.14 (small confounding effect) to 0.39 (medium confounding effect).

5.2.1. True Model

In most cases, the relative bias is acceptable for the true model, except when the mediation effect is small, but the confounding effect is medium. When the mediation effect is small ($a = b = 0.14$, $a \cdot b = 0.0196$) and the confounding effect is medium (0.39), the relative bias of the mediation effect is close to 1. Under these extreme conditions, the confounder plays a dominating role in predicting the dependent variable η_Y compared to the mediator. When the true model is fit to the data, the relative bias in the estimates of the mediation effect is acceptable.

5.2.2. Misspecified Measurement Model and No Measurement Model for Mediator

In this subsection, we examine the impact of two types of measurement model misspecifications on mediation effect estimates: (1) a misspecified measurement model and (2) an ignored measurement model using a standardized total score. These two conditions are grouped together because they both involve errors in the measurement model, though they differ in severity. Structuring them within the same subsection allows for clearer interpretation and direct comparison of their effects on parameter estimates.

For both types of errors in the measurement model of the latent mediator, the relative bias increases notably, particularly when no confounders ($\beta = 0$) are present or the confounding effect is small ($\beta = 0.14$). The relative bias ranges approximately from −0.25 to −0.75.

Using a standardized total score as the "mediator" to estimate the mediation effects further reduces the accuracy of the estimates, leading to a severe underestimation of the mediation effect, with the relative bias exceeding −0.50. This bias is more severe than that caused by ignoring the cross-loading of the mediator factor.

5.2.3. No Measurement Model for Confounder

Ignoring the measurement model for the $\xi \rightarrow \eta_M$ confounder and using the standardized total score generally have minimal impact on the estimates of the mediation effect. The relative bias remains acceptable (i.e., between −0.1 and 0.1) in most cases, except when a medium confounding effect is coupled with a small mediation effect. However, when the measurement model for the confounder on the path $\eta_M \rightarrow \eta_Y$ is also ignored, the mediation effect is significantly underestimated, leading to severe negative relative bias ranging approximately −0.25 to −0.50.

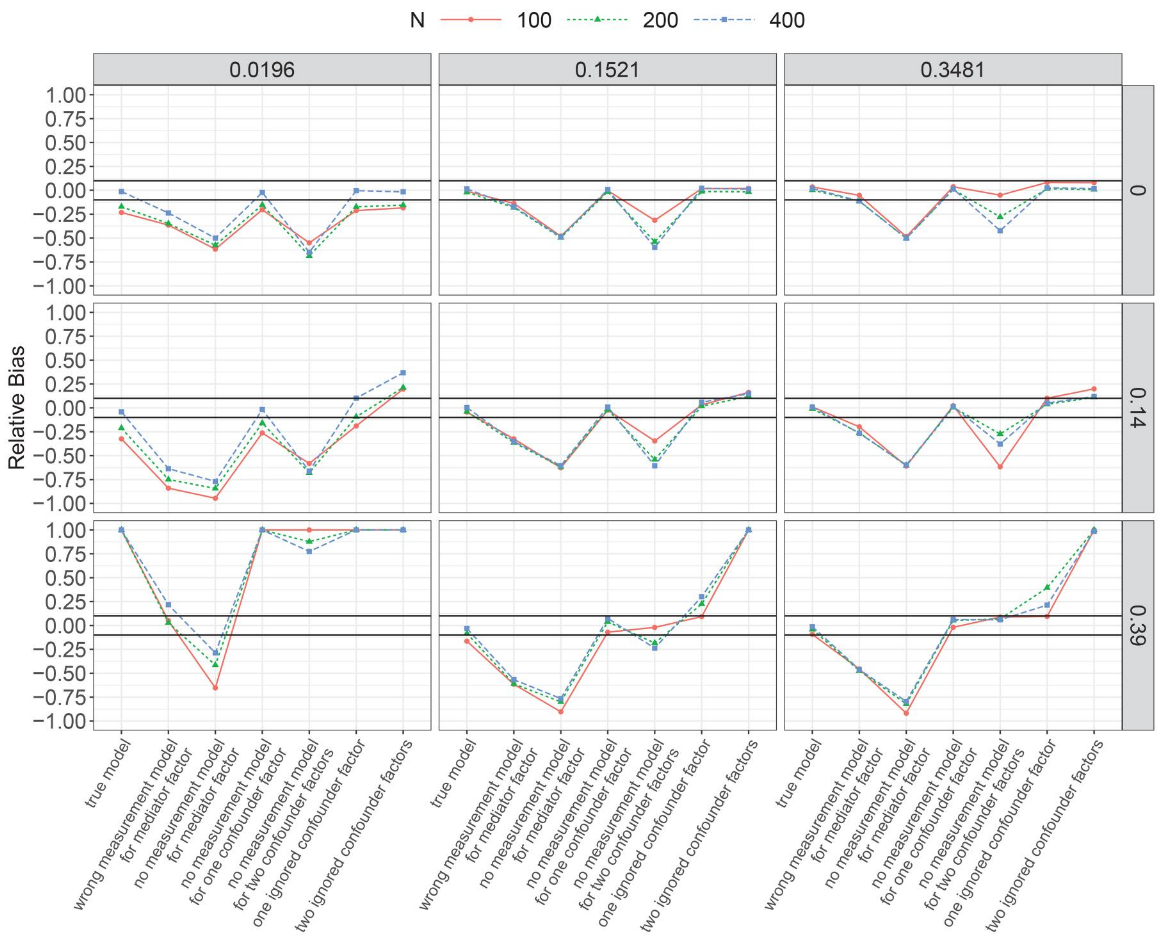

Figure 3. The relative bias of the mediation effect across seven model specifications under diffuse priors for both path coefficients and factor loadings. The columns in the grid represent different mediation effects, while the rows indicate the magnitude of the confounding effect. Each panel includes two black horizontal lines at 0.1 and −0.1, marking the acceptable range for relative bias.

5.2.4. Ignoring Confounders

Omitting confounders introduces substantial bias, especially when the confounding effect is medium-sized. In the current design, the path coefficients of the confounders are positive. When these confounders are ignored, the mediation effect is severely overestimated with a relative bias approaching 1. However, when the confounding effect is small ($\beta = 0.14$), the impact of ignoring the confounder is less severe, resulting in a relative bias of approximately 0.1–0.2.

Larger sample sizes reduce relative bias for the true model and lightly reduce the bias for the model that uses the standardized total scores for the mediator or the confounder.

5.2.5. Summary

With no misspecification, the Bayesian estimate of the indirect effect is accurate, except when the indirect effect is small, while the confounding effect is medium.

Misspecifying the mediator's measurement model significantly increases bias, particularly when no confounders are present or the confounding effect is small. Utilizing a standardized total score for the mediator exacerbates this bias, more so than ignoring cross-loadings. Neglecting the measurement model for confounders has minimal impact when the confounding effect is small; however, medium confounding coupled with a small mediation effect leads to unacceptable bias. Omitting the measurement model for confounders on the $\eta_M \rightarrow \eta_Y$ path results in severe underestimation of the mediation effect.

Finally, ignoring confounders leads to substantial overestimation of the mediation effect, especially when the confounding effect is medium. Larger sample sizes mitigate bias across all scenarios, with the greatest benefit observed under the correctly specified model.

5.3. Coverage Rates of the Credible Intervals

Figure 4 shows the coverage rate of the 95% credible intervals of the mediation effect. The two black horizontal reference lines are at 0.95 and 0.5. A coverage rate of around 0.95 is preferable. The columns of the grid represent different levels of the mediation effect ($a \cdot b = 0.0196$, 0.1521, and 0.3481), corresponding to small, medium, and large mediation effects, respectively. The rows reflect the magnitude of the coefficients for the paths of the confounders.

5.3.1. True Model

For the true model, the coverage rates are around 0.95 consistently for all conditions. This fact shows that the Bayesian estimation with diffuse priors estimates the credible interval accurately.

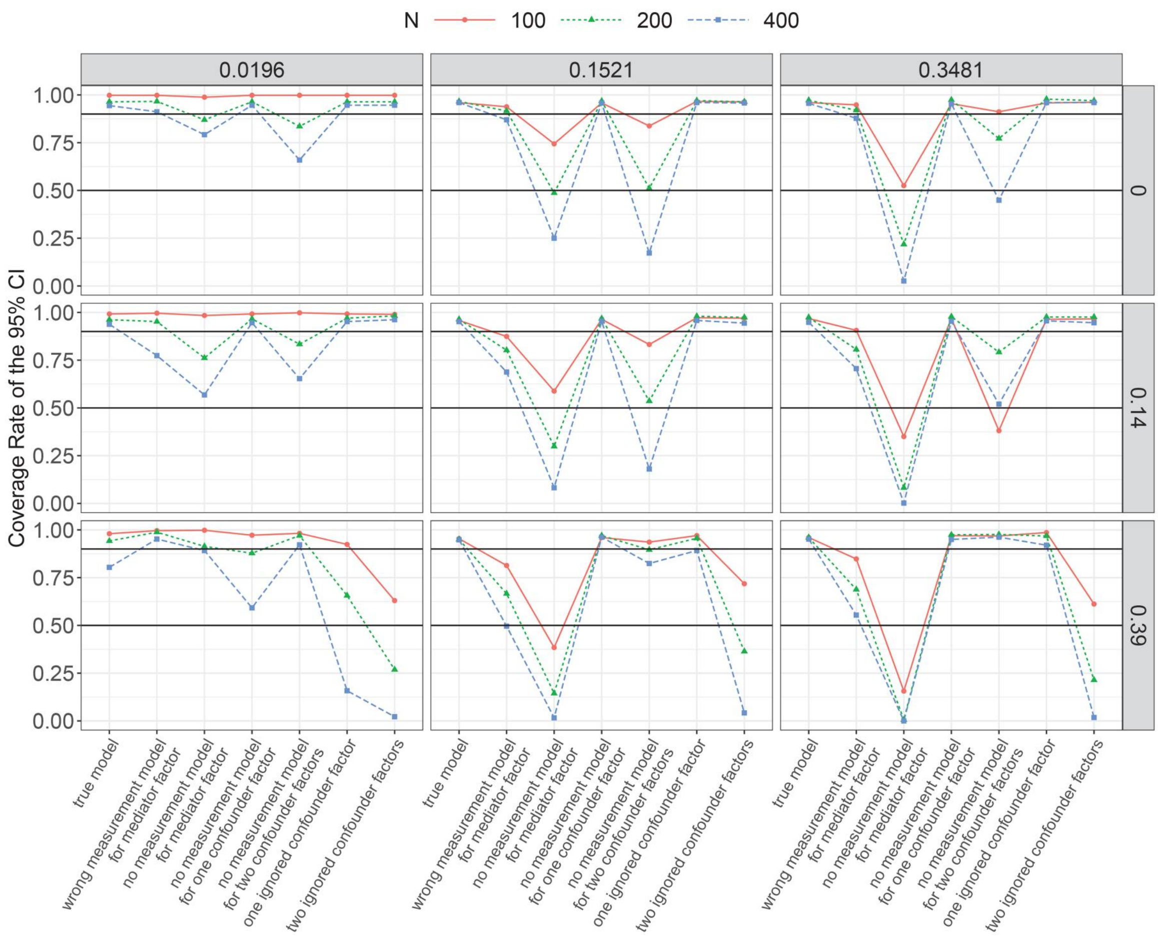

Figure 4. The coverage rates of the 95% Bayesian credible intervals of the mediation effect across seven model specifications when the diffuse priors are used. The columns in the grid represent different mediation effects, while the rows indicate the magnitude of the confounding effect. Each panel includes two black horizontal lines at 0.5 and 0.95. A coverage rate close to 0.95 is acceptable.

5.3.2. Misspecified Measurement Model and No Measurement Model for Mediator

Similar to the discussion on parameter estimate accuracy above, we examine the coverage rates for models with a misspecified measurement model and those without a measurement model for the mediator in the same subsection for comparison.

When the measurement model of the latent mediator is misspecified (i.e., ignoring the cross-loading), the coverage rates of the 95% credible interval fall below 0.95. As the confounding effect increases, the coverage rates deteriorate further.

When the measurement model is ignored, and the standardized total score is used, the coverage rates drop even more significantly. They can be as low as 50%.

5.3.3. No Measurement Model for Confounders

When the measurement structure of the confounders is ignored, standardized total scores are used instead. When only the confounder for the $\xi \rightarrow \eta_M$ path ignores the measurement model, the coverage rates remain close to 0.95. However, when the measurement models for both confounders are ignored, the coverage rates drop significantly. This issue is more severe when there is no true confounding effect or when the confounding effect is minor ($\eta = 0.14$). Interestingly, when the confounding effect is medium-sized ($\beta = 0.39$), the coverage rates return to around 0.95. This pattern highlights the complexity and varying influence of confounding effects on coverage rates in latent mediation analysis.

5.3.4. Ignoring Confounders

When the path coefficient of the confounders is small ($\beta = 0.14$), ignoring the confounders has little impact on the coverage rates of the mediation effect. However, when the coefficient of the confounder is medium-sized ($\beta = 0.39$), the coverage rates of the mediation effect drop severely.

Based on the results shown in Figure 4, we also observe that the coverage rates decrease with larger sample sizes. This occurs because larger sample sizes result in narrower credible intervals, which can lead to lower coverage rates.

5.3.5. Summary

For the true model, coverage rates consistently approximate 0.95 across all conditions, demonstrating the accuracy of Bayesian estimation with diffuse priors.

Misspecifying the mediator's measurement model, such as ignoring cross-loadings, reduces coverage rates, particularly when the confounding effect is medium-sized. Using standardized total scores for the mediator leads to even more significant declines, with coverage rates dropping to as low as 50%.

Ignoring the measurement model for confounders produces mixed results. When only the confounder for the $\xi \rightarrow \eta_M$ path lacks a measurement model, coverage rates remain close to 0.95. However, when both confounders use standardized scores, coverage rates drop significantly, especially with no or minor confounding effects. Medium confounding effects ($\beta = 0.39$) appear to mitigate this decline. Omitting confounders results in severe coverage rate reductions when the confounding effect is medium-sized, whereas minor effects have little impact.

5.4. Power for Detecting the Mediation Effect

Figure 5 demonstrates the ability of the Bayesian estimation method to detect the mediation effect. As in the other plots, the columns represent the magnitudes of the mediation effect, and the rows correspond to the values of the path coefficients for the confounders. The two horizontal lines represent a power of 0.90 and 0.50, respectively.

5.4.1. True Model

When fitting the true model, the power to detect the mediation effect increases as the mediation effect becomes larger. When the true mediation effect is 0.0196, the power can be as low as 0.5. For a medium-sized mediation effect of 0.1521, the power to detect the mediation effect exceeds 0.90 for sample sizes of 200 and above. With a large mediation effect of 0.3481, the power to detect the effect reaches 0.80, even for sample sizes as small as 100. Overall (and as would be expected), the power increases as the sample size becomes larger.

Moreover, the magnitude of the confounding effect also impacts the power to detect the mediation effect. As the path coefficient of the confounder increases from 0 to 0.39, the power drops slightly.

5.4.2. Misspecified Measurement Model and No Measurement Model for Mediator

For both the misspecified measurement model (i.e., omitting a cross-loading) and the no measurement model for the mediator (i.e., use the standardized total score), the power to detect the mediation effect is lower than in the true model. For a small mediation effect, the power remains

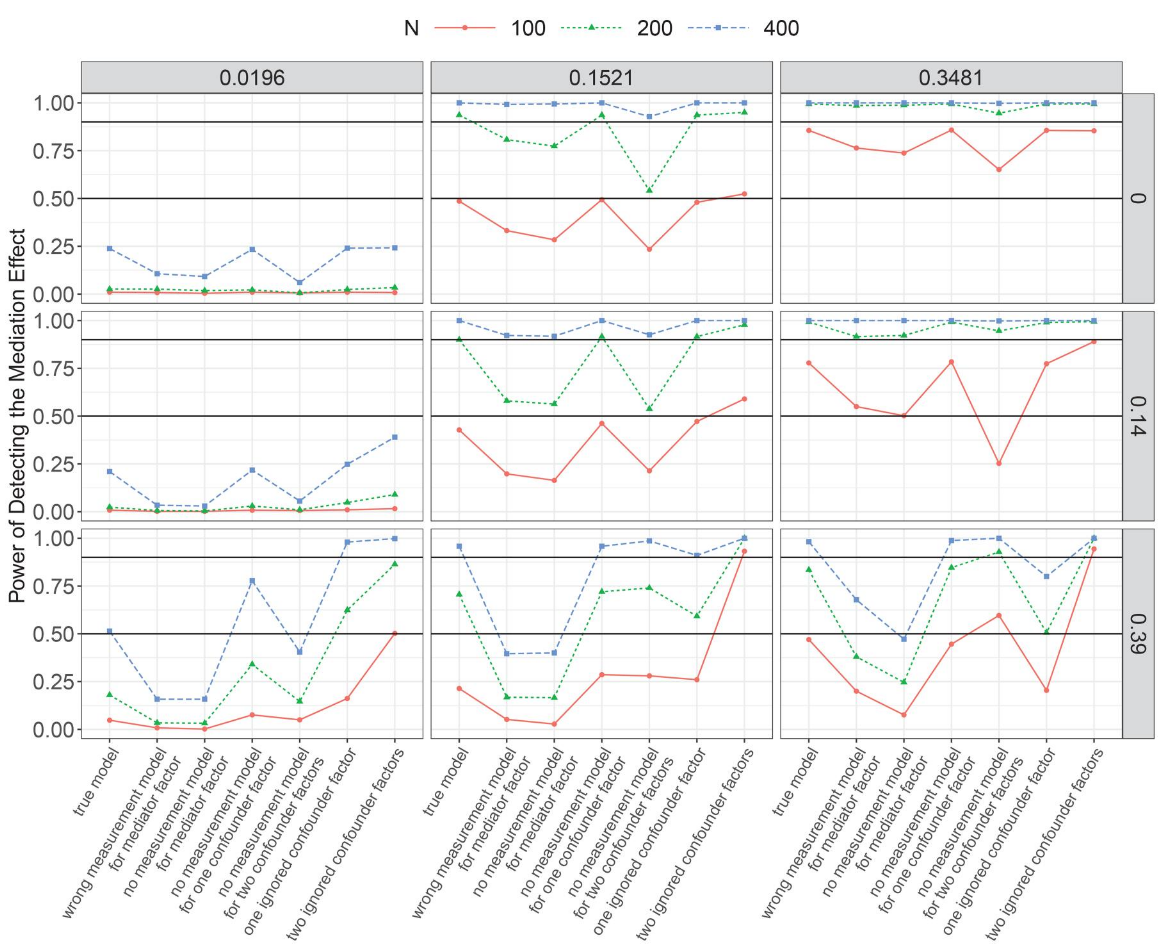

Figure 5. The power to detect the true mediation effect under diffuse priors. The grid's columns represent different mediation effects, while the rows correspond to varying magnitudes of the confounding effect. Each panel includes two black horizontal lines indicating power thresholds of 0.90 and 0.50, respectively.

below 0.25, even with a large sample size of 400. The power to detect a medium-sized mediation effect reaches 0.90 with a sample size of 200 and greater, provided there is no true confounder ($\beta = 0$). The power is even higher for a large mediation effect (i.e., 0.3481), indicating that the chance of detecting the mediation effect can still be high with medium or large mediation effects. As expected, the power increases with larger sample sizes and greater true mediation effects but drops as the confounding effect becomes more substantial.

5.4.3. No Measurement Model for Confounders

The power to detect the mediation effect decreases significantly when the measurement model of the confounders is ignored. This impact is more notable when the mediation effect is small or medium. As the confounding effect increases, the power decreases further, especially when the sample size is small and the confounding effect is large. However, a larger sample size can mitigate the effect of ignoring confounders and maintain relatively high power to detect the mediation effect.

5.4.4. Ignoring Confounders

When the confounding effect is null or small ($\beta = 0.14$), ignoring confounders has little impact on the power to detect the mediation effect. However, when the confounding effect is large, ignoring the confounders inflates the power.

To further understand this phenomenon, we refer back to the relative bias in Figure 3. When confounders are ignored, the covariation due to confounders is misattributed to the estimated mediation path, inflating the estimates of the mediation effect in proportion to the level of confounding effect under consideration.

5.4.5. Summary

Across all misspecified models, the power to detect the mediation effect suffers most from the misspecification of the measurement model for the latent mediator. The power also decreases due to the misspecification of the measurement model for the latent confounder. The impact of ignoring the latent confounder is mixed and depends on whether the mediation effect estimates are overestimated or underestimated.

A large sample size consistently promotes the power to detect the mediation effect, as well as the sign of the mediation effect.

5.5. Root Mean Square Error

Figure 6 demonstrates the RMSE for different model specifications. As in the other plots, the columns are the levels of the mediation effect, and the rows are the magnitudes of the path coefficient of the confounders. Each of the lines represents a sample size condition.

The RMSEA decreases as the sample size increases and the true mediation effect decreases. Additionally, the RMSE becomes larger as the confounding effect grows.

Among the seven models, ignoring confounders and using standardized total scores for latent mediators results in larger RMSEs compared to other models.

5.6. Prior Sensitivity Analysis

In the above analysis, we evaluated the accuracy of point estimates, the coverage of the posterior credible interval, and the power of each model to detect the mediation effect, focusing on diffuse priors. In empirical data analysis, weakly informative priors are often adopted to improve convergence for complex models while balancing the incorporation of prior knowledge with maintaining objectivity in inference. Therefore, we aim to assess how the accuracy and level of informativeness of the prior impact the performance of Bayesian estimation of the latent mediation model and how prior specification interacts with model misspecification.

In the current study, we varied the normal priors for the path coefficient and the factor loading parameters. We set the variance of the normal prior at $0.5 \times \text{true}$ to mimic the weakly informative prior. By adjusting the center of the normal distribution, we manipulated the level of accuracy of the priors. We considered three levels of accuracy: with the center of the normal prior set at the true value, $\sqrt{0.1 \times \text{true}}$ greater than the true value, and $2\sqrt{0.1 \times \text{true}}$ greater than the true value. These three weakly informative priors are called Weakly Informative Accurate (WI-Acc), Weakly Informative Inaccurate with the center deviated by $\sqrt{0.1 \times \text{true}}$ from the true value (WI-InAcc-1SD), and Weakly Informative Inaccurate with the center deviated by $2\sqrt{0.1 \times \text{true}}$ from the true value (WI-InAcc-2SD). Although these three priors share the same level of precision, their accuracy differs.

Our analyses indicated that the impact of weakly informative priors is not substantial for conditions with sample sizes of 200 and 400. However, it significantly influenced the results when the sample size was small, such as 100. Therefore, we present the results for $n = 100$ for brevity.

5.6.1. Accuracy of Parameter Estimates

Figure 7 shows the relative bias in the estimates of the mediation effect. The rows and columns of the grid represent the magnitude of the mediation effect and the levels of the path coefficients of the confounders, respectively. In each panel, the horizontal lines at −0.1 and 0.1 indicate the acceptable range for relative bias. Each spaghetti plot highlights a different prior.

5.6.1.1. True Model. For the true model, the three weakly informative priors and diffuse priors lead to similarly small relative biases when the mediation effect is medium or greater. Relative bias varies across priors primarily when the mediation effect is as small as 0.0196. Among the four

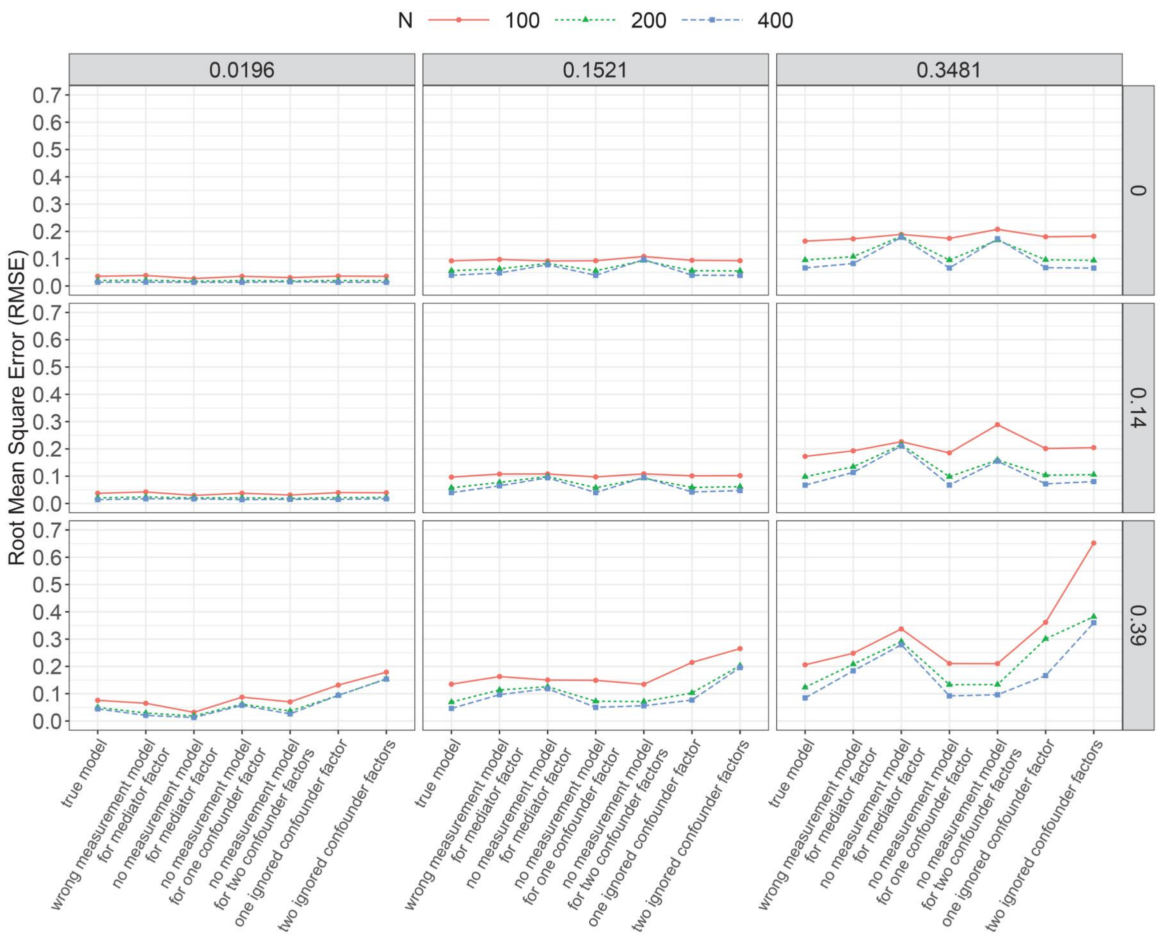

Figure 6. The root mean square error (RMSE) of the mediation (indirect) effect under diffuse priors. The grid's columns represent varying mediation effects and the rows correspond to different magnitudes of the confounding effect. Each spaghetti line in a plot represents the RMSE of a corresponding sample size.

priors, the weakly informative inaccurate prior with the center 2 SD away from the true value (WI-InAcc-2SD) causes severe bias, with a relative bias greater than 0.5.

5.6.1.2. Misspecified Measurement Model and No Measurement Model for Mediator. When the measurement model for the mediator is misspecified, all four priors underestimated the mediation effect with similar relative bias with mediation effect medium (0.1521) or large (0.3481). The bias was approximately −0.10 with no confounding effect, −0.25 with small confounding effects, and −0.50 with medium-sized confounding effects. When the mediation effect is small, its estimates become more sensitive to prior specification. Both the diffuse prior and weakly accurate prior led to an underestimation of the mediation effect. As the prior center shifted to the right, the relative bias decreased (i.e., WI-InAcc-1SD), and with a more extreme rightward shift (i.e., WI-InAcc-2SD), the estimates began to overestimate the mediation effect.

With no measurement model for the mediator, the standardized total score was used, leading to a severe underestimation of the mediation effect due to measurement error in the total score. The relative bias exceeds −0.50 when using the diffuse prior or the weakly informative accurate prior (WI-Acc) are used. However, weakly inaccurate priors with centers greater than the true value partially mitigate this underestimation of the mediation effect caused by the measurement error.

5.6.1.3. No Measurement Model for Confounders. When the standardized total score is used for the confounder on the path $\xi \rightarrow \eta_M$, the relative bias shows a mixed pattern depending on the levels of the confounding effect and the magnitude of the mediation effect. The distinction among the four priors is minimal when the mediation effect is medium or large. However, when the mediation effect is small (0.0196), there is a noticeable distinction among priors. The diffuse prior and the weakly informative accurate prior underestimate the mediation effect. In contrast, the two weakly informative inaccurate priors (WI-InAcc-1SD and WI-InAcc-2SD) overestimate the mediation effect, especially when the path coefficient of the confounders is small. When both confounders use the standardized total score, the mediation effect is underestimated across all priors, particularly when the path coefficients of the confounders are small.

5.6.1.4. Ignoring the Confounder. When confounders are ignored, the relative bias exceeds acceptable thresholds ($> 10\%$), particularly when the confounding effect is medium (0.39).

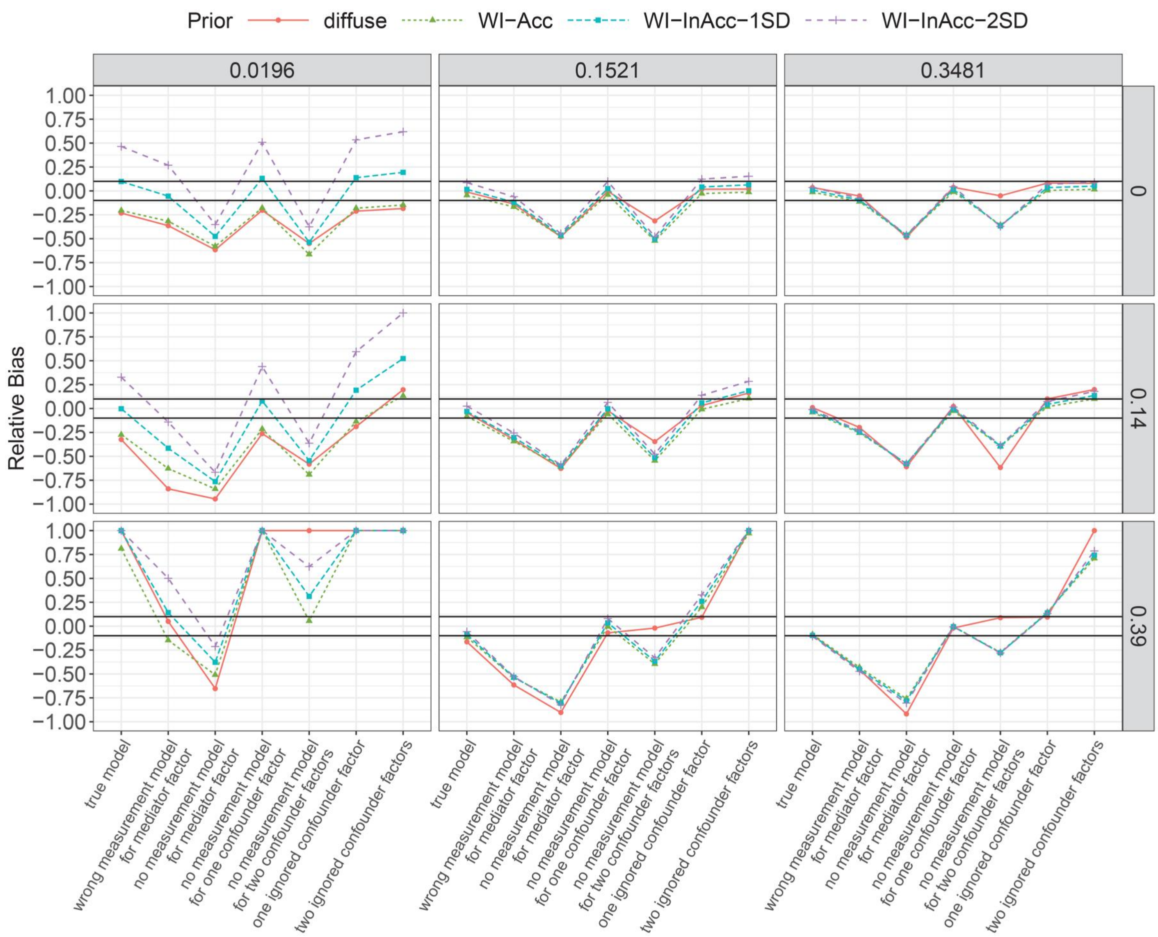

Figure 7. The relative bias of the mediation effect estimates across various prior specifications with a sample size of 100. The grid's columns represent different mediation effects, while the rows correspond to varying magnitudes of the confounding effect. The two black horizontal lines indicate the acceptable range for relative bias, set at 0.1 and −0.1. Each spaghetti line represents a specific prior specification.

When the mediation effect is small (0.0196), notable differences emerge among the four types of priors. With diffuse priors or weakly informative accurate priors (WI-Acc), the relative bias remains less severe and within an acceptable range (−10% to 10%). However, as the center of the weakly informative priors shifts further to the right, mediation effects become increasingly overestimated. Specifically, under the WI-Acc-1SD prior, the relative bias reaches approximately 0.25, and with the WI-InAcc-2SD prior, it exceeds 0.50.

In summary, the analysis demonstrates that both prior selection and model specification play a critical role in influencing relative bias in mediation effect estimates. For the true model, all priors yield similar results for medium or large mediation effects; however, weakly informative inaccurate priors result in severe bias for small effects. When the mediator's measurement model is misspecified, measurement error leads to underestimation of the mediation effect, though weakly inaccurate priors help mitigate this bias. For confounders, the use of standardized total scores produces mixed biases based on the mediation and confounding effects, with severe underestimation observed when both confounders lack adequate measurement models. Ignoring confounders causes significant biases, especially with diffuse priors, while weakly inaccurate priors partially reduce this bias. These findings highlight the necessity of precise prior selection and accurate model specification to achieve robust and reliable inference in Bayesian mediation analysis.

5.6.2. Coverage Rates of Credible Intervals

Figure 8 illustrates the coverage rates of the 95% credible interval, representing the proportion of replications in which the credible interval covers the true mediation effect. As an index of the validity of the credible interval estimates, a coverage rate of around 0.95 is desired. In each panel of the plots, the top horizontal line in black indicates a coverage rate of 0.95.

Based on the results, the priors have little impact on the coverage rates, except in scenarios where the total scores are used for the latent confounders or the confounders are ignored and the path coefficient of the confounders is medium (i.e., $\beta = 0.39$).

When using the total score for the confounders, using the diffuse prior leads to a slightly higher coverage rate.

5.6.3. "Power" for Detecting Mediation Effect

Figure 9 illustrates the power to detect the mediation effect across different prior specifications, magnitudes of

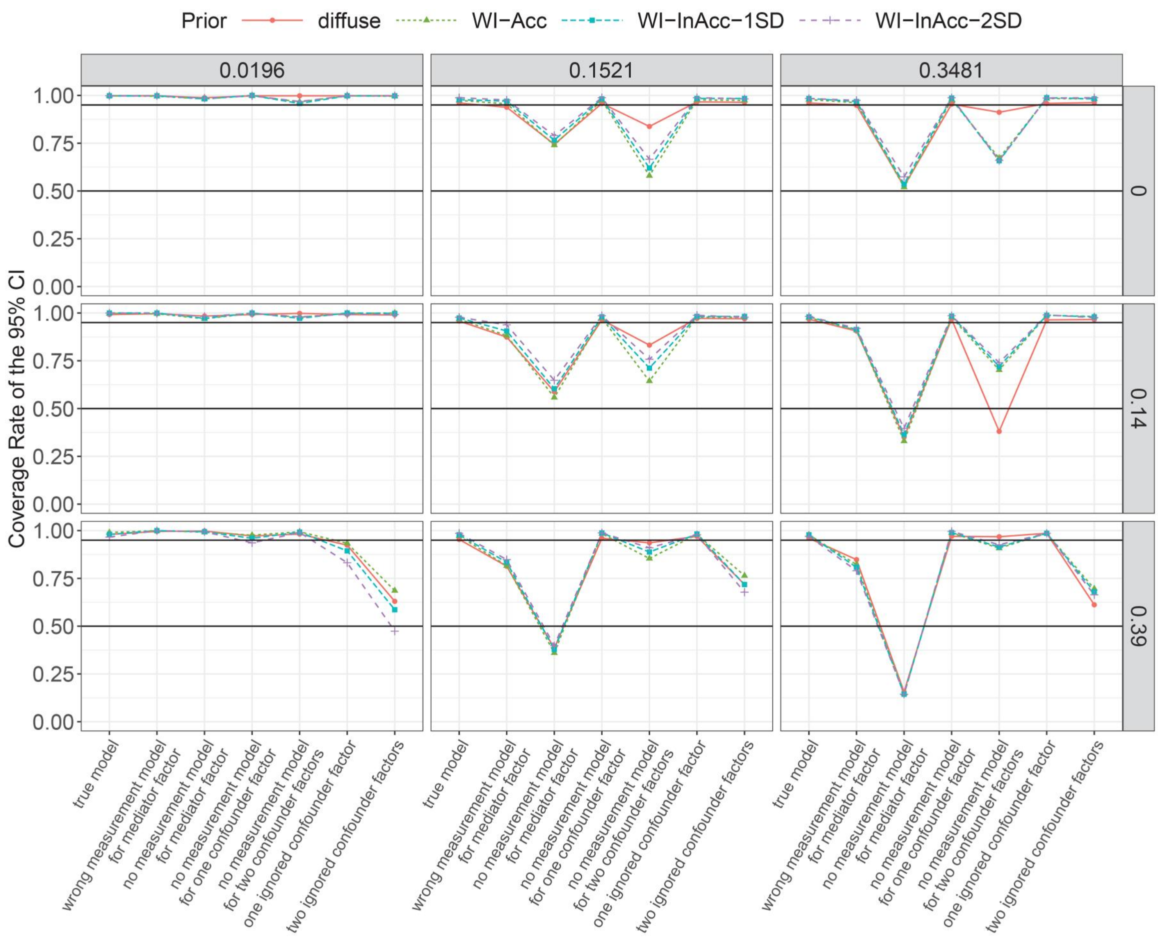

Figure 8. The coverage rates of the 95% Bayesian credible intervals with a sample size 100. The grid's columns represent different mediation effects, while the rows correspond to varying magnitudes of the confounding effect. The two black horizontal lines represent the coverage rate 0.50 and 0.95. Each spaghetti line represents a specific prior specification.

mediation effects, and levels of confounding effect when the sample size is 100. The columns represent the magnitudes of the mediation effect ($a \cdot b = 0.0196, 0.1521, 0.3481$), while the rows indicate the levels of the path coefficients of confounders ($\beta = 0, 0.14, 0.39$). The plots compare the power across four different prior specifications: diffuse prior, Weakly Informative Accurate (WI-Acc), Weakly Informative Inaccurate with 1 SD deviation (WI-InAcc-1SD), and Weakly Informative Inaccurate with 2 SD deviation (WI-InAcc-2SD),

For the true model, the power to detect the mediation effect increases with its magnitude. For small mediation effects (0.0196), the power is low across all priors. For medium (i.e., 0.1521) mediation effects, the power exceeds 0.60 for weakly informative priors and reaches 0.90 for the larger mediation effect (0.3481) when the sample size is 100.

The diffuse prior leads a lower power than the weakly informative priors (WI-Acc, WI-InAcc-1SD, and WI-InAcc-2SD), especially for medium and large mediation effects. When the mediation effect is large, all models have large power to detect it, except when the mediator model is misspecified and the confounding is large. For small mediation effects, the difference in power among priors is less pronounced, with all priors performing similarly poorly.

As the confounding effect increases from $\beta = 0$ to $\beta = 0.39$, the power to detect the mediation effect decreases slightly across all priors. When the confounding effect is small ($\beta = 0.14$), the power remains relatively high, but it drops noticeably when the confounding effect is medium ($\beta = 0.39$).

The presence of model misspecifications, such as ignoring the cross-loading of the latent mediator or confounders, reduces the power across all levels of mediation and confounding effects.

In sum, the power to detect the mediation effect is influenced by its magnitude, the level of the confounding effect, and the specification of priors. Although weakly informative inaccurate priors (WI-InAcc-1SD and WI-InAcc-2SD) can mitigate some biases, they also exhibit higher power in detecting mediation effects, particularly for small mediation effects.

5.6.4. Root Mean Square Error

The RMSE for conditions with a sample size of 100 is presented in Figure 10.

For all prior conditions, the RMSE increases as the mediation effect amplifies and as the path coefficient of the confounders becomes larger. With the diffuse prior, the RMSE

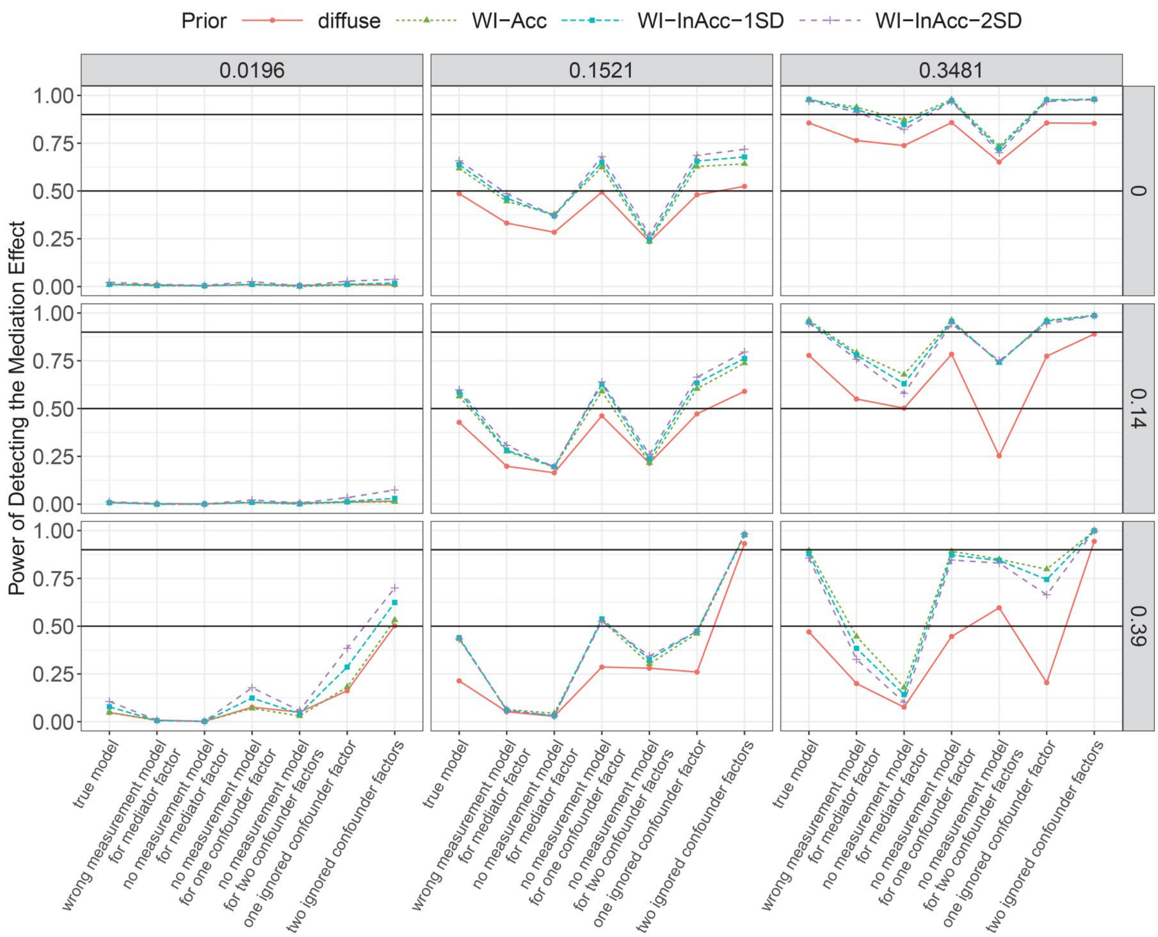

Figure 9. Power for detecting the mediation effect with a sample size 100. The grid's columns represent different mediation effects, while the rows correspond to varying magnitudes of the confounding effect. The two black horizontal lines represent the power 0.50 and 0.90. Each spaghetti line represents a specific prior specification.

is larger than with weakly informative priors, and this distinction becomes more apparent for larger mediation effects and confounder path coefficients.

6. Concluding Remarks

As mediation analysis among latent variables continues to gain popularity, it is vital to thoroughly understand the performance and impact of specification errors. Including the measurement model for latent variables aims to enhance the accurate understanding of these latent constructs. However, incorporating latent variables also increases the complexity of a model. Accurate inference of the mediation effect relies on two key assumptions: first, there are no unmeasured confounders; second, the measurement model is correctly specified. Unfortunately, these assumptions are often violated in practice. It is challenging to measure and control all relevant confounders, and the measurement model of latent variables can be misspecified. Furthermore, confounders could be latent variables, and their measurement models could be misspecified or omitted.

6.1. Summary of the Findings

In this study, we investigated the performance of the Bayesian estimation method in estimating the mediation effect, focusing on how this performance varies with violations of the model assumptions. Specifically, we examined the impact of incorrect or ignored measurement models for the mediator, ignored measurement models for the confounder, and omission of confounders. Furthermore, we assessed how model misspecifications interact with prior specifications and impact the performance of Bayesian methods in estimating the mediation effect. We considered both diffuse priors and weakly informative priors with different levels of accuracy to mimic practical implementation.

For the correctly specified (true) model, Bayesian estimation can accurately estimate the mediation effect with an acceptable relative bias, especially with medium or large mediation effect. The coverage rates were consistently around 0.95, indicating reliable Bayesian estimation with diffuse priors. The power to detect the mediation effect increased with larger mediation effects and sample sizes, but diffuse priors generally resulted in lower power compared to weakly informative priors. RMSE increased with larger mediation and confounding effects, but larger sample sizes reduced RMSE, improving the accuracy of parameter estimates.

With misspecified measurement models for the latent mediator, the relative bias increased notably, especially when there were no confounders or when the confounding effect

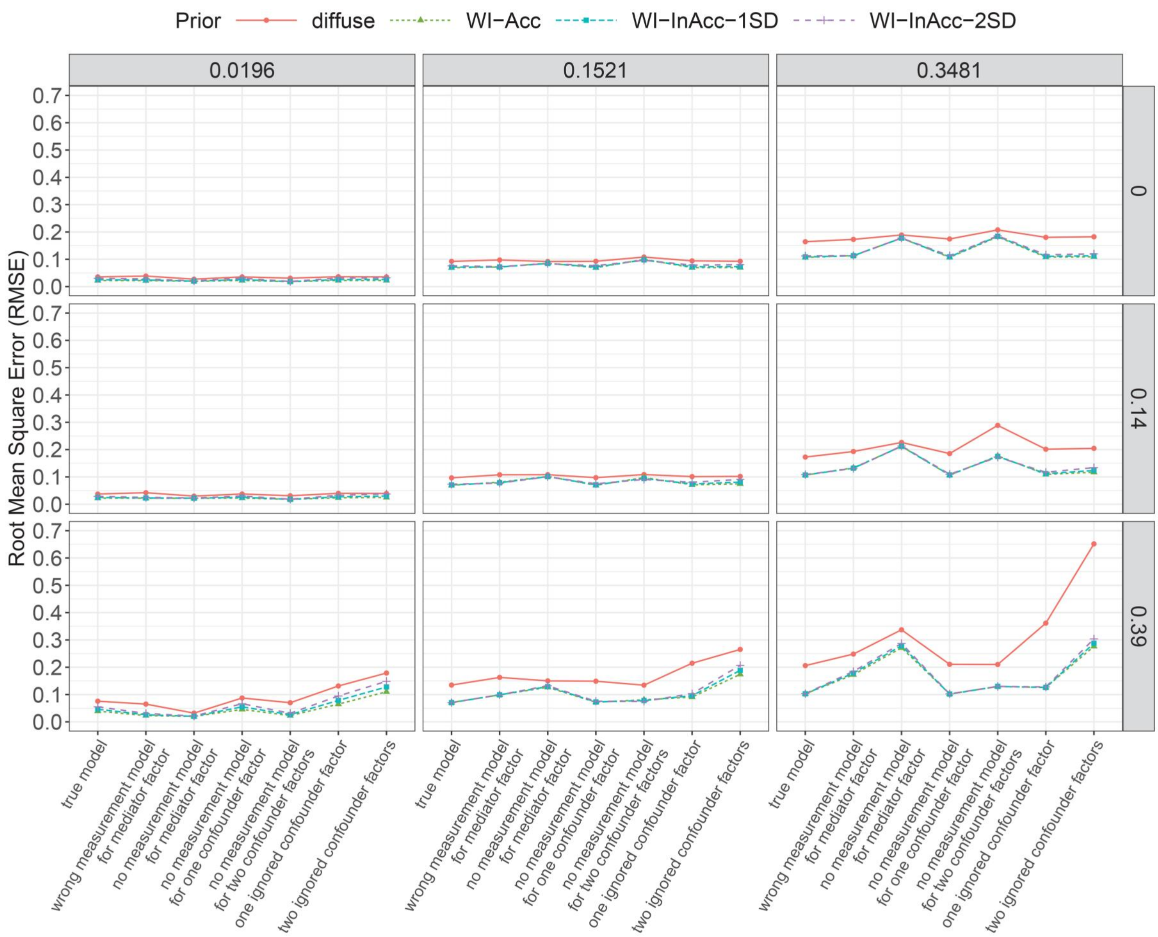

Figure 10. Root mean square error (RMSE) with a sample size 100. The grid's columns represent different mediation effects, while the rows correspond to varying magnitudes of the confounding effect. Each spaghetti line represents a specific prior specification.

was small. Using standardized total scores further reduced accuracy, leading to unacceptable bias. The coverage rates dropped significantly, particularly with no or minor confounders. The power to detect the mediation effect also suffered, especially with small mediation effects, although larger sample sizes and higher mediation effects improved power. RMSE increased with misspecified measurement models, especially with larger mediation and confounding effects.

Ignoring the measurement model for the $\xi \rightarrow \eta_M$ confounder generally had a minimal impact on the estimates of the mediation effect. However, ignoring both confounders' measurement models led to significant underestimation and severe negative relative bias. The coverage rates were significantly reduced, except when the confounding effect was medium in size. The power decreased when the measurement model for the confounder was ignored, particularly with medium-sized confounding effects. RMSE increased with ignored measurement models, especially with larger mediation and confounding effects.

Ignoring confounders overestimated the mediation effect, particularly with medium-sized confounding effects. Larger sample sizes slightly mitigated this bias. Coverage rates were significantly affected, particularly with medium-sized confounding effects, and larger sample sizes reduced coverage rates due to narrower credible intervals. Ignoring confounders inflated the power with large confounding effects, aligning with the observed overestimation of mediation effects. RMSE increased when confounders were ignored, especially with larger mediation and confounding effects.

6.2. Suggestions for Practitioners of Latent Mediation Analysis

Our simulation study emphasizes the vital importance of accurate model specification, thoughtful prior selection, and sufficient sample sizes in Bayesian latent mediation analysis to achieve robust and reliable results.

6.2.1. Accurate Model Specification of the Mediator

Correctly specifying measurement models for mediators is crucial. Ignoring measurement structures (e.g., cross-loadings) or using simplified representations (e.g., standardized total scores) can introduce significant bias, reduce coverage rates, and weaken statistical power. Researchers should avoid using aggregate scores and instead incorporate the full measurement structure of latent mediators and confounders. To achieve this, they should rigorously evaluate the theoretical basis of their models and ensure that the measurement models align with the data to prevent misspecifications.

To assess the fit of the measurement model of the mediator, researchers can use fit indices commonly applied in structural equation modeling (such as BRMSEA, BCFI, and BTLI) along with the general Bayesian fit indices (such as DIC, BIC, and PPP).

6.2.2. Addressing Confounders

Based on our findings, we would recommend researchers avoid using standardized total scores for latent confounders in the mediator–dependent variable path; instead, incorporate the full measurement structure. For unmeasured confounders, consider alternative approaches, such as modeling correlated residuals among indicators, as suggested by Zhang and Wang (2024). However, their effectiveness in latent mediation analysis should be assessed through sensitivity analyses.

6.2.3. Careful Prior Selection

For correctly specified models with medium or large mediation effects, both diffuse and weakly informative priors yield similar estimates, making them safe choices in these scenarios. However, when the mediation effect is small, prior selection becomes critical.

We acknowledge that in applied research the true parameter value is unknown and that researchers must rely on prior knowledge to inform their priors. Therefore, careful consideration of prior selection is essential to minimize bias, particularly when measurement error is present or when model assumptions may be violated.

6.2.4. Adequate Sample Sizes

When the model is correctly specified, larger sample sizes consistently enhance the accuracy of parameter estimates, increase statistical power, and reduce RMSE. For medium to large mediation effects, a sample size of 200 or more is generally sufficient to detect the mediation effect. However, detecting smaller mediation effects requires a substantially larger sample size.

By following these guidelines, researchers can mitigate common sources of bias and strengthen the validity and reliability of their findings in Bayesian latent mediation analysis.

6.3. Limitations and Future Directions of Research

This study has several limitations that should be acknowledged. First, our investigation focused on a specific set of model misspecifications, including incorrect or ignored measurement models for the mediator, ignored measurement models for the confounder, and the omission of confounders. However, other types of misspecifications, such as those in the structural model or violations of the normality assumption, were not considered. Future research should consider a broader array of model misspecifications, including those in the structural model, non-normal distributions of latent variables, and violations of other statistical assumptions. This will provide a more comprehensive understanding of the robustness of Bayesian estimation methods.

Second, this study focused on diffuse priors and weakly informative priors with varying levels of accuracy. Future research should expand the scope to include a broader range of prior distributions, such as highly informative priors, alternative diffuse priors, and empirically derived priors. Exploring these variations would provide deeper insight into the sensitivity of Bayesian estimates to the prior specifications and enhance the robustness of Bayesian latent mediation analysis.

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