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Parameterizing the LISREL Model as a Correlation Structure Model for More Efficient Parameter Estimates and More Powerful Statistical Tests

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ABSTRACT

Most methods for structural equation modeling (SEM) focused on the analysis of covariance matrices. However, "Historically, interesting psychological theories have been phrased in terms of correlation coefficients." This might be because data in social and behavioral sciences typically do not have predefined metrics. While proper methods for conducting correlation structure analysis have been developed, they emphasized on either how to get consistent standard errors of parameter estimates or how to ensure that the model-implied matrix remains to be a correlation matrix. Motivated by the fundamental needs for more efficient/accurate parameter estimates and greater power in conducting statistical tests, this article explores advantages of correlation structure analysis over its conventional covariance counterpart. Issues related to reparameterization and placement of parameters are discussed. A new concept is introduced for comparing efficiency/accuracy of parameter estimates that are not on the same scale. Via the analysis of many real datasets, meta results show that correlation structure analysis yields uniformly more accurate parameter estimates and more powerful statistical tests than its covariance-structure-analysis counterpart on parameters that are of substantive interests. The same pattern of results between the two model parameterizations is also found by Monte Carlo simulation. Issues related to correlation structure analysis and substantive elaboration of models that are not scale-invariant are discussed as well. The results are expected to promote technical and software developments of correlation structure analysis as well as its adoption in data analysis.

KEYWORDS

Correlation structure; efficiency of parameter estimates; meta comparison; signal-to-noise ratio; statistical power

1. Introduction

Data in social and behavioral sciences typically contain measurement errors. Structural equation modeling (SEM) is the most effective methodology for the analysis of such data. Method and software developments made SEM an easily accessible and versatile tool. However, measurements in social and behavioral sciences seldom have predefined metrics, which makes the values/sizes of the path coefficients in SEM lack of a direct substantive interpretation. A commonly used approach to avoid the issue of lack of metrics is standardized solutions so that the results are believed to be scale-free. Such a belief might deserve discussions from different perspectives (Yuan & Zhang, 2024). The interest of the current article is to study the statistical advantages of correlation structure analysis as compared to covariance structure analysis in SEM. In particular, we will show that, compared to covariance structure analysis, correlation structure analysis has the following properties: (1) it yields more efficient parameter estimates for both direct and indirect effects; (2) the Wald- or z-tests for both direct and indirect effects are statistically more powerful; (3) it generates the same consistent parameter estimates and asymptotically correct test statistics even if different scales for the manifest variables might have been used in different studies (e.g., average score, total score); and (4) there is no need to distinguish between sample correlation matrices and sample covariance matrices in conducting SEM analysis.

Most SEM methods are for covariance structure analysis (e.g., Yuan & Bentler, 2007). This is because classical SEM methods were so developed by assuming multivariate normality for the observed variables (e.g., Lawley & Maxwell, 1971; Jöreskog, 1969; Browne, 1974). Under such a setup, the results of normal-distribution-based maximum likelihood (NML) can be directly used for model and parameter inferences, including (1) the likelihood ratio statistic asymptotically follows a chi-square distribution, and (2) standard errors (SEs) for parameter estimates can be mechanically computed from the inverse of the information matrix. The wide implementation of the ML/NML method in SEM programs (e.g., EQS, LISREL, Mplus, lavaan) further pushed the studies and development for SEM with covariance structure models (e.g., Browne, 1984; Yuan & Bentler, 1998). Currently, covariance structure analysis is essentially the default method for structural equation modeling.

While proper methods for correlation structure analysis were developed (e.g., Bentler, 2007; Bentler & Savalei, 2010; Jennrich, 1974), researchers tend to stay away from dealing with sample correlation matrices. For example, for a dataset with only a sample correlation matrix being available, Jöreskog and Sörbom (1993, p. 39) provided "fictitious standard deviations" to avoid possible inconsistency results of analyzing a correlation matrix. Similarly, for the analysis of a correlation matrix of Worland et al. (1984), Kline (2011, pp. 283–284) also "assigned plausible standard deviations" to avoid the issues of analyzing a correlation matrix. Warnings against the use of correlation matrices (e.g., Krane & McDonald, 1978; Cudeck, 1989) might have contributed to such avoidance. Other substantive factors that have caused such avoidance are (I) there is a lack of research on the advantages of correlation structure analysis, and (II) software development and promotion for correlation structure analysis are behind technical development. Factor I can be a direct cause of factor II. In this article, to address factor I, we show that, by parameterizing a covariance structural model as a correlation structure model, the estimates of direct and indirect effects become more efficient in addition to providing a sound method for directly analyzing sample correlation matrices.

Bentler (2007) pointed out that psychologists tend to formulate theories or hypotheses via correlations[1] rather than covariances, due to lack of established metrics in measurements. He also noted that the adoption of correlation structure analysis is rather slow in SEM, although technical developments for correlation structure analysis were in place (Bentler, 2007; Bentler & Savalei, 2010) and limited studies on the performances of differently formulated test statistics in testing equality of correlation coefficients as well as the overall structure of a correlation model have been conducted, with emphasis on type I error controls with varying sample sizes and violation of distribution conditions (Steiger, 1980a, 1980b; Leung & Chan, 1997; Fouladi, 2000). The reason behind such a phenomenon might be because existing methodology developments for correlation structure analysis were mainly concerned with how to get asymptomatically correct SEs of parameter estimates as well as under what conditions the resulting model-implied covariance matrix remains to be a correlation matrix (Bentler & Savalei, 2010; Browne, 1982; Cudeck, 1989; Jamshidian & Bentler, 2000; Jennrich, 1974; Krane & McDonald, 1978; Shapiro & Browne, 1990). Our aim in the current article is not to reinforce these points nor to replicate the existing findings. Instead, we show that parameterizing a covariance structure model as a correlation structure model is not only well aligned with the conventional formulation of psychological theory but also provides more efficient parameter estimates and more powerful statistical test.

While the sample covariance matrix $\mathbf{S}$ and the sample correlation matrix $\mathbf{R}$ have systematically different properties, recent literature has pointed out that treating $\mathbf{R}$ as $\mathbf{S}$ in the ML or NML method for SEM still yields valid statistical inferences with respect to both overall model evaluation and null hypothesis testing for parameters of a standard LISREL model (Yuan et al., 2024). In particular, the z-statistics remain the same between working with $\mathbf{S}$ or $\mathbf{R}$. This is because parameter estimates and their SEs are proportionally affected when the manifest variables are rescaled by sample standard deviations or other constants, and the effects are cancelled in the z-statistics. However, Yuan et al. (2024) also noted that, unless $\theta = 0$ holds, SE of $\hat{\theta}$ by treating $\mathbf{R}$ as $\mathbf{S}$ may not be asymptotically correct, and the ratio $z = \hat{\theta}/\mathrm{SE}$ *depends on how the model is parameteried*, although the value of z *does not depend on whether data are standardized or not*. In the current article, by parametering a LISREL model as a correlation structure model, we show that estimates of path coefficients under the *LISREL-correlation* model become more efficient than those under the conventional *LISREL-covariance* model. In addition, the LISREL-correlation model also yields asymptotically correct SEs of parameter estimates when only a sample correlation matrix is available in data analysis.

Because of the complexity of the issues, we are unable to conduct the study analytically. Instead, we will conduct the study via meta comparison using real data and by Monte Carlo simulation. Also, sample correlations and covariances are on different scales, existing concepts such as bias, variances or mean-squared errors (MSE) are not applicable to comparing parameters estimates that are on different scales, we will introduce a new concept to facilitate the comparison. In addition, the formulation of a covariance structure model as a correlation structure model involves regularity conditions and technical details, we will discuss these issues via the LISREL model. Technical elements for reparameterizing the LISREL model as a correlation structure model are given in Section 2. Meta comparison via real datasets and models is presented in Section 3. A Monte Carlo study is conducted in Section 4 to reinforce the findings of the meta comparison. Conclusion, discussion and recommendations are provided in Section 5.

2. Correlation Structure Modeling via Covariance Structure Analysis

This section is to set up the foundation for carrying out the study and to provide an overview of the technical aspects behind the numerical results presented in the following sections. A concept closely related to model parameterization is scale-invariant models. For such models, we will first describe how to parameterize a covariance structure model as a correlation structure model. The relationship of parameters between the two parameterizations are discussed next in the framework of the LISREL model. We also discuss models that are not scale-invariant. Then we introduce a concept to facilitate the comparison of efficiency/accuracy of parameter estimates that may not be on the same scale. We will also have a subsection on reparameterizing the simple regression model, which facilitates our understanding of the effect of standardization on efficiency of parameter estimates

[1]The quoted statement in the abstract of this article is from Bentler (2007, p. 772).

and the z-test. Throughout the article, we will use $\mathbf{S}$ and $\mathbf{R}$ for sample covariance and correlation matrices and $\Sigma(\cdot)$ and $\mathbf{P}(\cdot)$ for the covariance and correlation structure models, respectively.

2.1. Model Reparameterization

Consider a population with p manifest variables represented by $\mathbf{v} = (v_1, v_2, ..., v_p)'$, with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ. Let $\Sigma(\boldsymbol{\theta}) = (\sigma_{ij}(\boldsymbol{\theta}))$ be a covariance structure model for $\mathbf{v}$, $\mathbf{D} = \text{diag}(d_1, d_2, ...d_p)$ with $d_j > 0$ be a diagonal matrix, and $\mathbf{v}^* = \mathbf{D}\mathbf{v}$ be a vector obtained by rescaling the variables in $\mathbf{v}$. The model $\Sigma(\boldsymbol{\theta})$ is *scale-invariant with respect to rescaling the manifest variables* if for any vector $\boldsymbol{\theta}$ of admissible parameters there exists another vector $\boldsymbol{\theta}^*$ of admissible parameters such that

$$\Sigma(\boldsymbol{\theta}^*) = \mathbf{D}\Sigma(\boldsymbol{\theta})\mathbf{D}. \tag{1}$$

That is, the covariance structure model that holds for $\mathbf{v}$ equally holds for $\mathbf{v}^*$ without changing the functional forms of any $\sigma_{ij}(\boldsymbol{\theta})$, only the values of the parameters need to be changed. Such a characterization was formally stated by Browne (1982) for covariance structure models. The issues of scale-invariance for exploratory factor models were discussed by Krane and McDonald (1978) and Swaminathan and Algina (1978). Yuan et al. (2024) noted that a standard LISREL model is scale-invariant if there is no non-zero-equality constraint on parameters beyond the need for model identification, and they provided the formulas for the elements of $\boldsymbol{\theta}^*$ as functions of $\boldsymbol{\theta}$ and $\mathbf{D}$. Browne (1982), Cudeck (1989) and Bentler (2007) gave examples of not scale-invariant models, where the models either contain equality constraints of parameters related to different variables or there exist non-zero values of parameters beyond the need for anchoring the scales of latent variables. Most models in our meta comparison in Section 3 are scale-invariant, while we also have results for models that are not scale-invariant.

Note that the covariance structure model $\Sigma(\boldsymbol{\theta})$ is to account for the relationship of the variables in $\mathbf{v}$ at the observed metrics. We would like to have a model parallel to $\Sigma(\boldsymbol{\theta})$ when the variables in $\mathbf{v}$ are standardized. Let $\mathbf{P}(\cdot)$ denote the p by p matrix whose off diagonal elements have the same functional forms as those of $\Sigma(\boldsymbol{\theta})$ while all its diagonal elements are 1.0. Corresponding to the covariance structural model $\Sigma(\boldsymbol{\theta})$, we define the correlation structure model $\mathbf{P}(\cdot)$ via a covariance structure representation

$$\Sigma_*(\boldsymbol{\theta}_*) = \Delta\mathbf{P}(\boldsymbol{\theta}_{*1})\Delta, \tag{2}$$

where $\Delta = \text{diag}(\delta_1, \delta_2, ..., \delta_p)$ is a diagonal matrix with $\delta_j > 0$, and $\boldsymbol{\theta}_* = (\boldsymbol{\theta}'_{*1}, \boldsymbol{\delta}')'$ with $\boldsymbol{\delta}' = (\delta_1, \delta_2, ..., \delta_p)$. If the covariance structure model $\Sigma(\boldsymbol{\theta})$ is scale-invariant, then for any vector $\boldsymbol{\theta}$ of admissible parameters there always exists a vector $\boldsymbol{\theta}_*$ of admissible parameters such that

$$\Sigma(\boldsymbol{\theta}) = \Sigma_*(\boldsymbol{\theta}_*). \tag{3}$$

Clearly, Equations (2) and (3) are a direct consequence of Equation (1) or a derived property of scale-invariant models. However, there are also categorical differences between Equation (1) and Equations (2) and (3). In Equation (1), the positions/placement of individual parameters remain the same between $\boldsymbol{\theta}^*$ and $\boldsymbol{\theta}$, only their values are accordingly changed. In Equations (2) and (3), because of the restrictions on the diagonal elements of $\mathbf{P}(\boldsymbol{\theta}_{*1})$, the functional forms of the elements of $\Sigma_*(\boldsymbol{\theta}_*)$ will need to change. That is, $\Sigma_*(\boldsymbol{\theta}_*)$ is different from $\Sigma(\boldsymbol{\theta})$ in both parameter values and placements of free parameters. In particular, as will be explained via the LISREL model in the next subsection, $\boldsymbol{\theta}_{*1}$ has fewer elements than $\boldsymbol{\theta}$ or some elements of $\boldsymbol{\theta}$ are no longer needed in $\boldsymbol{\theta}_{*1}$.

Equation (2) was first presented by Jennrich (1974) for exploratory factor analysis (EFA). The parameterization via Equation (2) was also recommended by Krane and McDonald (1978), Browne (1982), Cudeck (1989), and Bentler (2007). However, as in Jennrich (1974), these authors emphasized its utility in obtaining consistent SEs for parameter estimates of correlation structures rather than the benefits of more accurate parameter estimates and more powerful statistical tests in evaluating direct and indirect effects.

2.2. The LISREL Model

We next consider how $\boldsymbol{\theta}_{*1}$ is related to $\boldsymbol{\theta}$ when Equations (2) and (3) are formulated in the notation of the LISREL model, which is a specific representation of a general SEM model. Let $\mathbf{x}$ and $\mathbf{y}$ be vectors of indicators for the exogenous and endogenous latent vectors $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$, respectively. Then, with centralized data, the LISREL model is given by (Jöreskog & Sörbom, 1996, p. 2)

$$\mathbf{x} = \Lambda_x\boldsymbol{\xi} + \mathbf{e}_x, \quad \mathbf{y} = \Lambda_x\boldsymbol{\eta} + \mathbf{e}_y, \tag{4}$$

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \Gamma\boldsymbol{\xi} + \boldsymbol{\zeta}, \tag{5}$$

where Λ_x and Λ_y are factor loading matrices, $\mathbf{B}$ and Γ are matrices of path coefficients, $\mathbf{e}_x$ and $\mathbf{e}_y$ are measurement errors with covariance matrices $\Psi_x = (\psi^x_{jk})$ and $\Psi_y = (\psi^y_{jk})$, respectively. Two other covariance matrices in the LISREL model are $\Phi = \text{Cov}(\boldsymbol{\xi})$ and $\Psi_\zeta = \text{Cov}(\boldsymbol{\zeta})$. So there are a total of 8 matrices of parameters in a standard LISREL model. Note that our notations for error covariance matrices here are slightly different from those used in the LISREL software manual (Jöreskog & Sörbom, 1996) because θ and δ have been used for other purposes in this article.

Let $\mathbf{v} = (\mathbf{x}', \mathbf{y}')'$ and $\Sigma(\boldsymbol{\theta})$ be the covariance structure model for $\mathbf{v}$, with $\sigma_{ij}(\boldsymbol{\theta})$ being the covariance of v_i and v_j formulated according to Equations (4) and (5). A specific representation of $\Sigma(\boldsymbol{\theta})$ via the eight matrices of the LISREL model is given in Equation (1.4) of Jöreskog and Sörbom (1996, p. 3), with $\boldsymbol{\theta}$ consisting of the free parameters in the eight matrices. We refer to $\Sigma(\boldsymbol{\theta})$ as a LISREL-covariance model in this article. Corresponding to each LISREL-covariance model $\Sigma(\boldsymbol{\theta})$, there exists a LISREL-correlation model $\mathbf{P}(\boldsymbol{\theta}_{*1})$ according to Equation (2). Specifically, the off-diagonal elements of $\mathbf{P}(\boldsymbol{\theta}_{*1})$ have the same functional forms as those of the LISREL-covariance model $\Sigma(\boldsymbol{\theta})$ while $\text{diag}[\mathbf{P}(\boldsymbol{\theta}_{*1})] = \mathbf{I}$. However, for the correlation structure model, we still need to specify the placements of the parameters in $\boldsymbol{\theta}_{*1}$ with respect to the 8 matrices of the LISREL model. Note that the model $\Sigma_*(\boldsymbol{\theta}_*)$ has p standard deviation parameters $\delta_1, \delta_2, \cdots, \delta_p$ not belongs to $\boldsymbol{\theta}_{*1}$. At the same time, $\text{diag}[\mathbf{P}(\boldsymbol{\theta}_{*1})] = \mathbf{I}$ forces the elements of $\boldsymbol{\theta}_{*1}$ to satisfy p constraints, implying that there are p redundant elements for using the LISREL model to parameterize the correlation structure $\mathbf{P}(\cdot)$. Thus, we need to remove p parameters from the conventional LISREL-covariance model due to the p side conditions over the elements of $\boldsymbol{\theta}_{*1}$. A programmatic and also convenient approach is to let the diagonal elements of Ψ_x and Ψ_y of the LISREL-correlation model be 1.0 minus the true-score variance of the corresponding indicators rather than being freely estimated. For example, for the jth indicator in $\mathbf{x}$, $\psi^x_{jj} = 1 - (\Lambda_x \Phi \Lambda'_x)_{jj}$ is a function of the elements of Λ_x and Φ instead of being counted as an extra parameter. Then, if the model does not contain correlated errors, $\boldsymbol{\theta}_{*1}$ of the LISREL-correlation model consists of parameters only from six matrices. They are

$$\Lambda_x, \quad \Lambda_y, \quad \mathbf{B}, \quad \Gamma, \quad \Phi, \quad \text{and} \quad \Psi_\zeta. \tag{6}$$

For clarity, we will use $\boldsymbol{\theta}_1$ to denote the vector of free parameters in the above 6 matrices for the conventional LISREL-covariance model. We will call the parameterization with error variances being given by one minus the true score variances a *regularity condition* for correlation structural models. Thus, with the regularity condition, the placements and functionality of the parameters in $\boldsymbol{\theta}_{*1}$ with the LISREL-correlation model are the same as those of $\boldsymbol{\theta}_1$ with the LISREL-covariance model but for standardized variables. If error covariances exist, they appear in the off-diagonal elements of Ψ_x or Ψ_y of the LISREL-covariance model. Then $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_{*1}$ will also contain the off-diagonal elements of Ψ_x and Ψ_y in additional to the parameters in the six matrices in (6). Such a specification guarantees that $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_{*1}$ have the same number of elements and so do $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_*$.

Letting measurement-error variances be functions of other parameters rather than themselves being freely estimated was first used by Jennrich (1974) for obtaining consistent SEs of factor loading estimates in EFA. Bentler and Savalei (2010) noted that the same techniques can be applied to LISREL and Bentler-Weeks models (Bentler & Weeks, 1980) in their development of asymptotic theory for correlation structure analysis. Other developments for obtaining consistent SEs of parameter estimates with standardized variables include Lawley and Maxwell (1971), Browne (1982), and Jamshidian and Bentler (2000). Note that the LISREL-correlation model formulated by Equation (2) is a special parameterization of a covariance structure model. Thus, all the techniques for covariance structure analysis can be directly applied (e.g., fit indices, rescaled test statistics, and treatment of incomplete data) to LISREL-correlation models. In particular, Equation (3) always hold as long as the LISREL-covariance model $\Sigma(\boldsymbol{\theta})$ is scale-invariant. Equation (3) also holds at the sample level as well even when the model is not correctly specified. Thus, for scale-invariant models, the likelihood ratio statistic $T_{ml} = (N-1)F_{ml}$ will attain the same values whether the model is parameterized as $\Sigma(\boldsymbol{\theta})$ or $\Sigma_*(\boldsymbol{\theta}_*)$.

Note that the LISREL platform for SEM also permits variables without-measurement-errors to serve as exogenous or endogenous variables in the structure model. For such exogenous variables, their variances will be fixed at 1.0 in $\mathbf{P}(\boldsymbol{\theta}_{*1})$ and their standard deviations be on the diagonal of Δ in Equation (2). For endogenous variables without measurement errors, their prediction error variances will be one minus the model-implied variances while their standard deviations are on the diagonal of Δ in Equation (2). We will also call such models LISREL-correlation models, and use $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_{*1}$ to denote the vectors of parameters that appear in the off-diagonal elements of $\Sigma(\boldsymbol{\theta})$ and $\mathbf{P}(\boldsymbol{\theta}_{*1})$, respectively.

2.3. Not-Scale-Invariant Models

We have discussed reparameterizations of scale-invariant LISREL-covariance models as LISREL-correlation models. For a covariance structure model that is not scale-invariant, we can continue to parameterize it as a correlation structure model via Equation (2). The placements of parameters between the two parameterizations are the same as for scale-invariant models. That is, measurement-error variances of LISREL-correlation models are functions of the parameters of the other six matrices while the other elements of the model (including fixed values or constraints over parameters) are specified the same as for the LISREL-covariance models. However, Equation (3) no-longer holds. This is because not-scale-invariant models typically contain substantive hypotheses regarding the strength of relationships among the variables, and such constraints for parameters of the model with the observed metrics do not necessarily equally hold for parameters of the model with the standardized metrics, and vice versa. The consequences are: (1) the model implied covariance matrix $\Sigma(\hat{\boldsymbol{\theta}})$ is different from $\Sigma_*(\hat{\boldsymbol{\theta}}_*)$, and (2) the likelihood ratio statistics T_{ml} are different between the two parameterizations, although the two models have the same number of free parameters. But each parameterization can enjoy its own substantive interpretation.

A classical example of substantive constraint is parallel tests, which assume that all the items have equal true score variances and equal error variances (Allen & Yen, 1979). Such an assumption may not equally hold in the observed and standardized metrics. The two parameterizations respectively model and/or test the two sets of assumptions. One can use the values of T_{ml} or other model selection criteria to endorse one parameterization over the other, as will be further discussed in the concluding section.

We next illustrate the interpretation of parameters of not-scale-invariant models by regression analysis with two predictors. Without constraints, the model

$$y = b_1 x_1 + b_2 x_2 + e \tag{7}$$

is scale-invariant in the sense that the R^2 is the same whether we work with the raw variables or standardized variables. Least-squares (LS) estimates of regression coefficients with standardized variables can be obtained by proper scale-transformation of the LS estimates for (7). In contrast, the model

$$y = b_1 x_1 + b_1 x_2 + e, \tag{8}$$

is not scale-invariant because of the constraint $b_2 = b_1$. However, the model

$$z_y = \beta_1 z_{x_1} + \beta_1 z_{x_2} + \varepsilon \tag{9}$$

is also substantively interesting, where the zs are standardized scores. In particular, Equation (8) assumes that the regression coefficient of x_1 equals that of x_2, implying that y increases b_1 units when either x_1 or x_2 increases by 1.0 while holding the other predictor constant. In contrast, Equation (9) implies that y increases $\beta_1 s_y$ units when x_1 increases s_{x_1} units or when x_2 increases s_{x_2} units while holding the other predictor constant, where the s represents the sample SD. Clearly, unless $s_{x1} = s_{x_2}$, the implications of Equations (8) and (9) are different and they cannot be equally plausible. Also, even if $\sigma_{x_1} = \sigma_{x_2}$ in the population, the probability for Equations (8) and (9) to have the same R-square value is zero. Thus, it is not a surprise for a SEM model to have different values of T_{ml} under different metrics.

2.4. Signal-to-Noise Ratio (SNR)

The previous subsections focused on parameterizing a covariance structure model as a correlation structure model or vice versa. We would like to compare the efficiency/accuracy of parameter estimates across the two parameterizations. However, parameter estimates $\hat{\theta}$ and $\hat{\theta}_*$ as well as their population counterparts are not on the same scales even if the model is scale-invariant, and the commonly used concepts such as bias, variance and MSE are not applicable to evaluating parameter estimates that are on different scales. In this subsection, we introduce a concept, termed as *signal-to-noise ratio* (SNR), to facilitate the comparison of estimates by different parameterizations, on different metrics or for the analysis of data without predefined metrics. Note that we do not intend to compare measurement-error variances $\hat{\psi}^x$ and $\hat{\psi}^y$ against the standard deviations $\hat{\delta}^x$ and $\hat{\delta}^y$, because they serve different purposes in the LISREL model. Rather, we use the SNR to compare the efficiency of $\hat{\boldsymbol{\theta}}_1$ against that of $\hat{\boldsymbol{\theta}}_{*1}$, simply because they are rescale of each other, and $\boldsymbol{\theta}_1 = \mathbf{0}$ if and only if $\boldsymbol{\theta}_{*1} = \mathbf{0}$.

Let $\hat{\theta}$ be an unbiased or consistent estimate of a parameter θ based on a sample of size N. Suppose there exist a sequence of positive numbers c_N and a constant ω_θ^2 such that

$$c_N(\hat{\theta} - \theta) \text{ converges in distribution to } N(0, \omega_\theta^2), \tag{10}$$

as N increases. We call ω_θ the standard deviation (SD) of $\hat{\theta}$ and define the SNR of $\hat{\theta}$ as

$$\tau = \frac{\theta}{\omega_\theta}. \tag{11}$$

Clearly, the SNR is a measure for the size of the systematic part of the estimate $\hat{\theta}$ relative to its error, after accounting for the effect of the sample size. A greater value of SNR implies less relative errors in $\hat{\theta}$ and consequently a more efficient/accurate estimate. The SNR can be regarded as a generalization of $\tau = \mu/\sigma$ for the sample mean $\hat{\theta} = \bar{x}$, and it applies to any parameter estimate. When $\hat{\theta}$ and $\hat{\theta}_*$ share the same sequence c_N and have the same expected or limiting value, or the limiting value of $\hat{\theta}_*$ is a rescale of that of $\hat{\theta}$ in a modeling context, the ratio of the two SNRs becomes the square root of the (asymptotic) relative efficiency of the two estimators as in definition 10.1.16 of Casella and Berger (2002, p. 476). Thus, among sets of estimators that share the same c_N and with proportional limiting values, the SNR serves as a standing alone measure of efficiency of a parameter estimate.

The sequence c_N in (10) is termed as the normalizing constant by Casella and Berger (2002, p. 470). Under a set of regularity conditions,[2] $c_N = N^{1/2}$, which holds for maximum likelihood estimates, pseudo maximum likelihood estimates, robust M-estimates, and estimates that are functions of sample moments (see e.g., Casella & Berger, 2002; Ferguson, 1996; Gourieroux et al., 1984; Huber, 1981). Since the focus of this article is the NML estimates of the LISREL model under different parameterizations, we will assume $c_N = N^{1/2}$ from now on.

The value of the τ in Equation (11) depends on the model, the population distribution of the sample as well as the estimation method[3] used to obtain $\hat{\theta}$. However, τ does not depend on the sample size N asymptotically or depends little on N with finite samples. This is because (asymptotic) standard errors of parameter estimates are inversely proportional to $N^{1/2}$, and $\omega_\theta = \sqrt{N}$SE effectively removes the dependence on N. In addition, SNR does not depend on *a priori* chosen scales of the involved variables for all sensible estimation methods. For example, if θ is the regression coefficient of y on x and $\hat{\theta}$ is obtained by least squares or a robust method, then the corresponding τ remains the same if we change x to $d_x x$ and/or y to $d_y y$, where $d_x > 0$ and $d_y > 0$ are non-stochastic constant. Thus, unlike variance or MSE, the SNR can be used to compare estimates by different researchers who might have scaled variables or parameterized the model differently. However, the value of the SNR does change if d_x or d_y are data-dependent and the estimation method takes the randomness of d_x and d_y into account. We will show analytically in the following subsection that the SNR for the estimate of a simple regression coefficient with standardized variables is greater than that with raw variables.

For a LISREL-covariance model $\Sigma(\boldsymbol{\theta})$, if individual manifest indicators are rescaled and we continue to estimate the

[2]Let $f(\mathbf{x}, \boldsymbol{\theta})$ be the probability density function corresponding to the population distribution of the data/sample. A key component of the regularity conditions is that the support, which is defined as the set of $\mathbf{x}$ such that $f(\mathbf{x}, \boldsymbol{\theta}) > 0$, is mathematically independent of the parameters $\boldsymbol{\theta}$. When this condition is not met, examples exist for $c_N = N$ and $c_N(\hat{\theta} - \theta)$ is not asymptotic normal for a maximum likelihood estimate $\hat{\theta}$ (see e.g., Ferguson, 1996, p. 96 & 132; Lehmann & Casella, 1998, p. 439).

[3]An example for SNR to depend on the estimation method and the underlying population distribution is to consider a sample from a symmetric population distribution with a finite variance. Let $\hat{\theta}$ and $\tilde{\theta}$ be the sample mean and sample median, respectively. Then, it depends on the shape of the population distribution for $\hat{\theta}$ to have a greater or smaller SNR than that of $\tilde{\theta}$ (see Casella & Berger 2002, p. 484; Ferguson, 1996, pp. 91–92).

model by NML and treat the rescaled data as normally distributed, then the SNRs for all the parameter estimates in $\hat{\boldsymbol{\theta}}$ remain the same even if the scaling factors are data dependent (Yuan et al., 2024). In parallel, for a LISREL-correlation model $\Sigma_*(\boldsymbol{\theta}_*)$ as in Equation (2), the SNRs for all the parameter estimates in $\hat{\boldsymbol{\theta}}_*$ also remain the same regardless whether the scaling factors are data-dependent or not, as will be further explained in Section 2.6. However, except for the correlations among exogenous latent variables whose scales are anchored by $\mathrm{Var}(\xi) = 1.0$, the SNRs for estimates of the other direct and indirect effects between the two parameterizations of the LISREL model are different, as will be shown in Sections 3 and 4.

Because ω_θ (the SD of $\hat{\theta}$) is consistently estimated by $\hat{\omega}_\theta = \sqrt{N}\mathrm{SE}$, we can consistently estimate τ by $\hat{\tau} = z/\sqrt{N}$, where $z = \hat{\theta}/\mathrm{SE}$. Thus, a greater z corresponds to a greater $\hat{\tau}$ and vice versa. Among estimates of θ by different methods, the α-level confidence interval (CI) for θ based on an estimate with a greater SNR will be narrower if the estimates are made equal by rescaling the related variables. Thus, statistical tests or inference via CIs based on parameter estimates with greater SNRs will be statistically more powerful. However, the concept SNR primarily intends to serve as a measure of efficiency/accuracy of an estimator rather than serving as a test statistic. In particular, SNR is a quantity that does not depend on sample size. In contrast, the p-value, power of a test statistic, and non-centrality parameters (NCP) of a t-, z- or χ^2-distributions for testing H_0: $\theta = 0$ typically depend on sample size.

The concept SNR, as defined in Equation (11), is essentially a standardized difference between θ and 0, which was initially introduced by Yuan and Fang (2023) and further discussed in Yuan and Zhang (2024) for modeling data without predefined metrics. A generalization for SNR from a single parameter to multiple parameters was noted in Yuan and Fang (2023) via the Mahalanobis distance between vectors $\boldsymbol{\theta}$ and $\mathbf{0}$ weighted by the precision matrix of $\sqrt{N}\hat{\boldsymbol{\theta}}$. Formally, the multivariate SNR of $\hat{\boldsymbol{\theta}}$ is given by

$$\tau^2 = \boldsymbol{\theta}'\Omega_\theta^{-1}\boldsymbol{\theta}, \tag{12}$$

where $\boldsymbol{\theta} = E(\hat{\boldsymbol{\theta}})$ or is the probability limit of $\hat{\boldsymbol{\theta}}$ as the sample size N increases, and Ω_θ is the asymptotic variance-covariance matrix of $\sqrt{N}\hat{\boldsymbol{\theta}}$. A consistent estimate of the τ^2 in Equation (12) is obtained by $\hat{\tau}^2 = T_w/N$, where T_w is the Wald statistic for testing

$$H_0 : \boldsymbol{\theta} = \mathbf{0}. \tag{13}$$

As expected, the multivariate τ^2 does not depend on the scales of the involved variables nor sample size. Among estimates of parameters that account for the relationship of the same set of variables of the LISREL model, the estimates with greater univariate or multivariate SNRs contain less errors and thus are more efficient. If the interest is to test the H_0 in Equation (13), then $N\tau^2$ is the noncentrality parameter (NCP) of the chi-square distribution that T_w approximately follows. Thus, a greater τ^2 directly implies a greater power value in testing (13). In Sections 3 and 4, we will use $\hat{\tau}^2$ as a measure of efficiency/accuracy of parameter estimates when comparing the advantage of LISREL-correlation models against LISREL-covariance models.

We have emphasized that the θ in Equation (11) is the expected value of $\hat{\theta}$ or its probability limit. In SEM or other contexts of statistical modeling, it might be an impossible task to find a correct model. Then, conditional on the substantive model, $\hat{\theta}$ and θ are the sample value and the population counterpart of a generic parameter of the model that we choose to use to approximate the population of interest. Suppose the population covariance matrix $\Sigma = E(\mathbf{S})$ is available (e.g., in Monte Carlo studies), and the value of θ is obtained by fitting the model $\Sigma(\boldsymbol{\theta})$ to Σ via the same estimation method as $\hat{\theta}$ was obtained in fitting $\Sigma(\boldsymbol{\theta})$ to $\mathbf{S}$. Then it can be shown that θ is the probability limit of $\hat{\theta}$ (see e.g., Shapiro, 1983; Yuan, Hayashi & Bentler, 2007). The same result holds for θ_* and $\hat{\theta}_*$ with correlation structure models. Thus, the concept SNR truthfully reflects the efficiency of the estimate of a parameter of the substantive model we choose to use regardless of whether the model is correctly specified or not.

2.5. SNR with the Simple Regression Coefficient

Consider the regression model

$$y = \alpha + \beta x + e,$$

where $e \sim N(0, \sigma^2)$. The predictor x can be random or nonstochastic. We will only consider the case with random predictor here and assume $x \sim N(\mu_x, \sigma_x^2)$, and e and x are independent. This puts the regression model as a special case of SEM by the NML method. With a sample of size N, let $s_{xx} = s_x^2$ and s_{xy} be the sample variance of x and covariance of x with y, respectively. Then, it follows from the so-called conditional variance identity (see e.g., Casella & Berger, 2002, p. 167) that, for the LS estimate of β, there exist

$$\begin{aligned}\hat{\beta} = s_{xy}/s_{xx}, \quad \mathrm{Var}\left(\sqrt{N}\hat{\beta}|x\right) &= \sigma^2/s_x^2,\\ \mathrm{Var}\left(\sqrt{N}\hat{\beta}\right) &= E(\sigma^2/s_x^2) + \mathrm{Var}\left[E\left(\sqrt{N}\hat{\beta}|x\right)\right]\\ &= \sigma^2 E(1/s_x^2), \quad \mathrm{Avar}\left(\sqrt{N}\hat{\beta}\right) = \sigma^2/\sigma_x^2,\end{aligned}$$

where Avar is the notation for asymptotic variance. Thus, the population or asymptotic SNR of $\hat{\beta}$ is

$$\tau_\beta = \sigma_x\beta/\sigma. \tag{14}$$

Let β_* be the standardized value of β and $\hat{\beta}_*$ be the corresponding estimate. Then β_* and $\hat{\beta}_*$ are just the Pearson correlation coefficients between y and x. It follows from Muirhead (1982, p. 159) that the asymptotic variance of $\hat{\beta}_*$ is

$$\mathrm{Avar}\left(\sqrt{N}\hat{\beta}_*\right) = (1 - \beta_*^2)^2.$$

Thus, with $\beta_* = \beta\sigma_x/\sigma_y$,

$$\begin{aligned}\tau_{\beta_*} &= \frac{\beta_*}{(1-\beta_*^2)} = \frac{\beta\sigma_x}{\sigma_y(1-\beta^2\sigma_{xx}/\sigma_{yy})} = \frac{\beta\sigma_x\sigma_y}{\sigma^2}\\ &= \frac{\beta\sigma_x}{\sigma}\frac{\sigma_y}{\sigma} = \tau_\beta(1+\beta^2\sigma_x^2/\sigma^2)^{1/2}.\end{aligned} \tag{15}$$

Equations (14) and (15) imply that $\tau_{\beta_*} > \tau_\beta$ whenever β is not zero. They also imply that testing for $\beta_* = 0$ is statistically more powerful than testing for $\beta = 0$, and the gain in power increases with $|\beta|$ and σ_x/σ. Note that $\beta_* = 0$ if and only if $\beta = 0$, and then $\tau_{\beta_*} = \tau_\beta$. Thus, both tests can control type I errors reasonably well for samples with medium to large N. For more reliable inference on correlations with finite samples, one should consider using Fisher's z-transformation (see Yuan & Bentler, 2000).

According to Equation (15), for normally distributed (x, y), the asymptotic efficiency of $\hat{\beta}_*$ over that of $\hat{\beta}$ is given by

$$(\tau_{\beta_*}/\tau_\beta)^2 = (1 + \beta^2\sigma_x^2/\sigma^2). \tag{16}$$

Results in Yuan and Bentler (2000) imply that (16) also holds when (x, y) follows an elliptical distribution, a pseudo[4] normal distribution, and a pseudo elliptical distribution. Thus, while our focus is on the NML estimates between the two parameterizations of the LISREL model, the finding is expected to hold within a larger class of nonnormal distributions, as to be further discussed in the concluding section.

With more than one predictor, the formulas of τ for regression coefficients with standardized variables become rather complicated. They become even more complicated with latent variable models. We will use meta comparison and Monte Carlo simulation to compare the empirical efficiency of parameter estimates under the two parameterizations.

2.6. Properties of Parameter Estimates and z-Statistics

Note that the parameter δ_j in Equation (2) is the model-implied SD of v_j, the jth manifest variable in $\mathbf{v}$. An interesting fact with correlation structure models is that, for the NML estimate $\hat{\delta}_j$, its SE is estimated by $\mathrm{SE}_j = \hat{\delta}_j/(2n)^{1/2}$ under the normality assumption. Thus, $z = (2n)^{1/2}$, and $\hat{\delta}_j$ is always significant at the .05 level whenever $n \geq 2$ or $N = (n+1) \geq 3$. This is also logically sound since we do not expect a standard deviation to be zero for a variable that is of substantive interest. In addition, this fact makes $\hat{\delta}_j$ and SE_j scale-equivariant and the corresponding z-statistic scale-invariant.

Because $\hat{\delta}_j$ absorbs the standard deviation of the jth observed variable v_j, the estimate $\hat{\boldsymbol{\theta}}_{*1}$ for the correlation structural model in Equation (2) remains the same even if we treat $\mathbf{R}$ as a sample covariance matrix under LISREL-correlation. But $\hat{\delta}_j = 1.0$ in such a case. Thus, $\hat{\boldsymbol{\theta}}_{*1}$ as well as their SEs are scale-invariant, and the z-statistics for all the elements of $\hat{\boldsymbol{\theta}}_{*1}$ are scale-invariant with respect to either stochastic- or constant-rescaling of the manifest variables in $\mathbf{v}$. Consequently, the SNRs for all the parameter estimates under LISREL-correlation remain the same even if we treat $\mathbf{R}$ as a sample covariance matrix in the analysis. This will be reflected by our results reported in the following section.

[4]A pseudo normal distribution has the same kurtosis as that of a normal distribution but can have arbitrary skewnesses, and a pseudo elliptical distribution is similarly defined (see Yuan & Bentler, 2000).

3. Meta Comparison

In this section we compare LISREL-correlation (CORR) against LISREL-covariance (COV) with respect to efficiency/accuracy of parameter estimates as measured by SNRs via the analyses of real data. We will present the results of 18 models based on 12 datasets. These datasets and models were not chosen to obtain particular results, nor were they selected so that a model has to fit a dataset well enough. Rather, they are from SEM textbooks, software demonstrations and authors who were willing to share their raw data with us. The datasets represent different areas where theoretical constructs are modeled for studies of substantive interests. For datasets that only sample covariance matrices are available, we will fit the structural models $\Sigma(\boldsymbol{\theta})$ and $\Sigma_*(\boldsymbol{\theta}_*)$ to the sample covariance/correlation matrices respectively by the NML method. When a raw dataset is available and it exhibits a significant departure from the normality assumption, as measured by Mardia's (1970) multivariate kurtosis, we will first apply a robust transformation (Yuan, Chan & Bentler, 2000) so that the transformed data approximately follow a multivariate normal distribution, which are next fitted by the two structure models by the NML method, respectively.

3.1. SNR, z-Statistic and Two Scaling Options

Because z-statistics are in default output of all statistical programs/software, and $\hat{\tau} = z/\sqrt{n}$, we will report the z-statistics without separately reporting the SNRs for individual parameter estimates. We will also report the z-statistics for indirect effects, where the standard error for each indirect-effect estimate is computed using the so-called delta-method (see Yuan & Schuster, 2013). As discussed in Section 2, comparison of the z-statistics between the two parameterizations of the LISREL model directly implies the relative efficiency of their parameter estimates as well as the corresponding advantage of statistical power.

We will use λ^x, λ^y, ϕ, γ, β, ψ^ζ to represent the parameters in the six matrices in (6). They are also the elements shared by $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_{*1}$, which also include error covariances ψ^x_{jk} and ψ^y_{jk} if the model contains such parameters. We will refer to these parameters as *shared parameters* in our following presentation, with focus on efficiency of their estimates. Nevertheless, we will report the z-statistics for all the individual parameters. To save space in tables, we will not include the symbols δ^x or δ^y. Instead, results under COV following ψ^x_{jj} and ψ^y_{jj} are for the estimates of error variances whereas those under CORR are for the estimates of SDs of the manifest variables. Similarly, results following $\tau^2_{\hat{\theta}}$ and $\tau^2_{\hat{\theta}_1}$ under COV are the multivariate SNRs for $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\theta}}_1$ while those under CORR are the corresponding SNRs for $\hat{\boldsymbol{\theta}}_*$ and $\hat{\boldsymbol{\theta}}_{*1}$, respectively.

For each model, we need to specify the scales of the latent variables. The scale of an exogenous latent variable can be fixed by letting one of its loadings equal 1.0 ($\lambda = 1.0$) or its variance equal 1.0 ($\phi = 1.0$). We will refer to the specification $\phi = 1.0$ as phi-anchor and $\lambda = 1.0$ as lambda-anchor. Gonzalez and Griffin (2001) showed that the values of z-statistics are different between the two anchor options. Results in Yuan and Zhang (2024) indicate that, among different options of lambda-anchor, estimates by letting the loading of the most reliable indicator equal 1.0 are more efficient. We include both phi-anchor and lambda-anchor to gauge the performances of the two parameterizations. However, the total number of different combinations of lambda-anchors can be large. For example, for a model with 1 exogenous and 3 endogenous latent variables with three indicators each, there are $3^4 = 81$ different options for lambda-anchors. For the results to be in a manageable scale, we will only specify the loading of the first indicator at 1.0 for lambda-anchor under both COV and CORR.

3.2. Model, Dataset and Result

Each model in this section is represented by a path diagram. With 18 models, there will be 18 tables for the results of individual parameter estimates, their SEs, and the resulting z-statistics as well as the multivariate SNRs τ^2_{θ}, $\tau^2_{\theta_*}$, $\tau^2_{\theta_1}$ and $\tau^2_{\theta_{*1}}$. To save space, the 18 tables (Tables A1 to A18) are put as supplementary material online https://www3.nd.edu/~kyuan/LISREL_correlation/TableA1_18.pdf. After discussing the results for each model, we will have a summary for the values of $\tau^2_{\theta_1}$ and $\tau^2_{\theta_{*1}}$, which is presented at the end of this section. For readers who are interested in verifying or replicating our results, the 12 sample covariance and correlation (for one dataset) matrices used in the meta comparison are available at https://www3.nd.edu/~kyuan/LISREL_correlation/Dataset_1to12.cov, where original contributors of the datasets are acknowledged. All the numerical results in this article were computed by Fisher-scoring algorithm and coded in SAS IML, which can be obtained from the authors.

3.2.1. Confirmatory Factor Model

Dataset 1. Holzinger and Swineford (1939) contain scores of 24 subtests of students from two middle schools. Jöreskog (1969) used 9 of the 24 scores from one school with $N = 145$ in illustrating his development of the NML method for SEM, and the dataset has been used repeatedly in many other publications and software demonstrations of the SEM methodology. According to Holzinger and Swineford (1939), the first three tests (visual perception, cubes, lozenges) were designed to measure *spatial* ability, the next three tests (paragraph comprehension, sentence completion, word meaning) were to measure *verbal* ability, and the last three tests (addition, counting dots, straight-curved capitals) aimed to measure a *speed* factor in performing tasks. The path diagram for a confirmatory factor model with the 9 variables is given in Figure 1, where x_1 to x_9 represent the test scores, and ξ_1, ξ_2 and ξ_3 represent the three

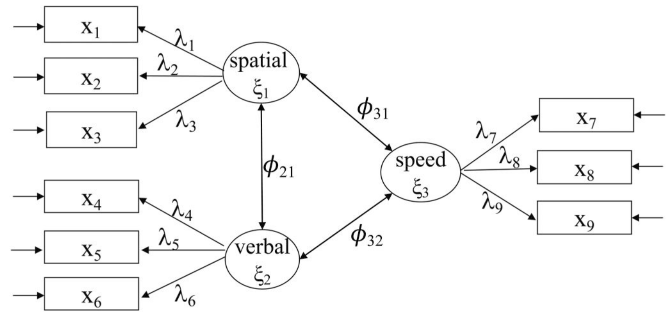

Figure 1. A confirmatory factor model with nine cognitive variables (Holzinger & Swineford, 1939; Jöreskog, 1969).

latent factors. The original nine variables were respectively divided by 6.0, 4.0, 8.0, 3.0, 4.0, 7.0, 23.0, 20.0, 36.0 so that they have comparable loading estimates in our presentation. Such a rescaling does not affect the efficiency of parameter estimates as measured by SNRs or z-statistics. The model has $q = 21$ parameters in $\boldsymbol{\theta}$ and $q_1 = 12$ shared parameters in $\boldsymbol{\theta}_1$.

Fitting the model in Figure 1 to Dataset 1 by normal-distribution-based maximum likelihood (NML) results in $T_{ml} = 51.187$, which is statistically significant at the level .05 when being referred to χ^2_{24}. However, fit indices (e.g., CFI = .942, Bentler, 1990; RMSEA = .089, Steiger & Lind, 1980) may suggest the model is practically acceptable. Table A1 contains four sets of parameter estimates, their SEs and z-statistics of the 21 parameters. The results on the left panel of the table are for phi-anchors of the three latent factors ($\phi_{11} = \phi_{22} = \phi_{33} = 1.0$), while those on the right panel are for lambda-anchors ($\lambda_1 = \lambda_4 = \lambda_7 = 1.0$). Except for the three factor correlations under phi-anchors on the left panel of the table, the z-statistics/SNRs for all the other parameters under CORR are uniformly greater than their counterparts under COV. The ratios of the two sets of z-statistics (SNRs) for the nine factor loadings range from 1.2 to 2.1 on the left panel while those for the 12 shared parameters range from 1.1 to 2.2 on the right panel. According to the values of the multivariate SNR, τ^2_{θ} and $\tau^2_{\theta_1}$ under CORR are 3.0 and 5.4 times those under COV on the left panel of Table A1, respectively; the ratios of the two multivariate SNRs are 2.7 and 4.1 on the right panel of the table.

3.2.2. Hierarchical Factor Model

According to Holzinger and Swineford (1939), the test scores for this dataset can also be modeled by a hierarchical factor model, as represented by the path diagram in Figure 2. Because the three first-order factors become dependent latent variables in the hierarchical model, we have changed the label for the indicators from x to y, although the two models are equivalent in the sense that they have the same model-implied covariance matrix.

Table A2 contains the results of fitting the model in Figure 2 to Dataset 1 by NML, where the layout of the table is essentially the same as Table A1, with the scale of ξ_1 being anchored by $\phi_{11} = 1.0$ on the left panel and by $\gamma_1 = 1.0$ on the right panel. On both panels of Table A2, the z-statistics/SNRs of the 21 parameter estimates by CORR are

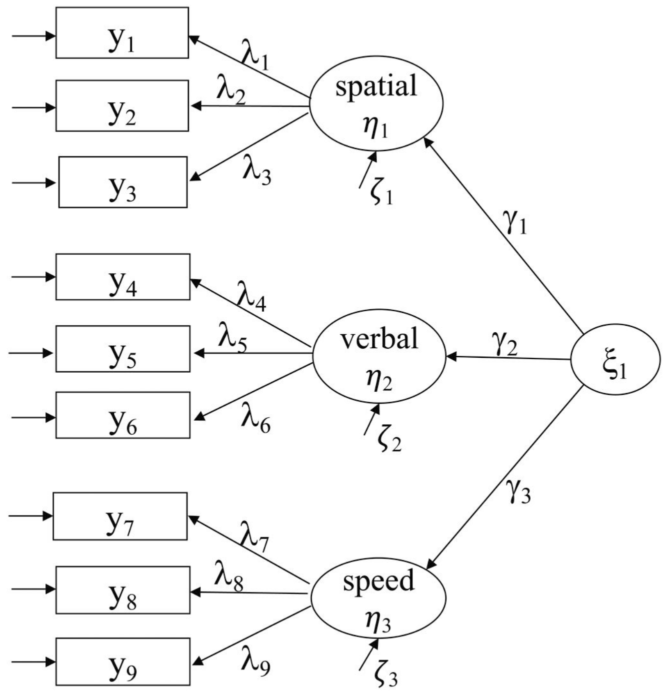

Figure 2. A hierarchical factor model with nine cognitive variables (Holzinger & Swineford, 1939).

uniformly greater than those by COV. The ratios of the z-statistics for the 12 shared parameters by CORR over those by COV on both sides of the table range from slightly above 1.0 to 1.5. The ratios of the multivariate SNR τ_θ^2 under CORR over that under COV are 3.0 on the left panel and 3.6 on the right panel of Table A2, while the ratios for the $\tau_{\theta_1}^2$ are 4.2 and 4.8, respectively, indicating that the $\hat{\theta}_{*1}$s are jointly 3 times more efficient than the $\hat{\theta}_1$s.

3.2.3. Mediation Model

Dataset 2. Table 7.3 of MacKinnon (2008, p. 186) contains an example that examines the mediation effect of coachs' attitude on players' intention to use steroids. Based on a sample of $N = 547$ high school football players, the table gives the sample correlation matrix together with sample standard deviations. The mediation model involves three latent variables and each is indicated by three manifest variables. According to MacKinnon, *Coachs' intolerance* was measured at time 1 and by questionnaire items: coach1—I have talked with at least one of my coaches about different ways to get stronger instead of using steroids, coach2—On my team there are rules against using steroids, and coach3—If I were caught using steroids, I would be in trouble with my coaches. The construct *perceived lack of severity* of steroid use was measured at time 2 and by questionnaire items: perception1—The bad effects of anabolic steroids go away as soon as you stop using them, perception2—Only a few people who use anabolic steroids ever have any harmful or unpleasant side effects, and perception3—Anabolic steroids are not dangerous if you use them only a few months each year. The construct *intention to use* steroids was measured at time 3 by items: intent1—I intend to try or use anabolic steroids, intent2—I would be willing to use anabolic steroids to know how it feels, and intent3—I am curious to try anabolic steroids. The path diagram representing the latent-variable model is given in Figure 3, which has 21 parameters in $\boldsymbol{\theta}$ and 12 shared parameters in $\boldsymbol{\theta}_1$. The model also has an indirect effect $\nu_1 = \gamma_{11}\beta_{21}$.

Fitting the model in Figure 3 to Dataset 2 by NML results in $T_{ml} = 29.111$ ($df = 24$), which correspond to a p-value of .216. Fit indices (CFI = .997, RMSEA = .020) also indicate that the model fits the data very well. Table A3 contains the results of parameter estimates for the mediation model in Figure 3, where the z-statistics by CORR are uniformly lager than their counterparts by COV, although the z-statistics for the parameter γ_{21} by different parameterizations are identical to the 3rd decimal place. This is because the value of $\hat{\gamma}_{21}$ is small, and then the z-statistics under CORR and COV tend to be close for tiny $\hat{\theta}$, as implied by the SNR in Equation (15) for the coefficient of a simple regression model.

For the 12 shared parameters and the indirect effect, the ratios of the individual z-statistics/SNRs by CORR over those by COV range from slightly above 1.0 to 1.7 on both panels of Table A3. According to the multivariate SNRs, the values of τ_θ^2 and $\tau_{\theta_1}^2$ by CORR over those by COV are 8.8

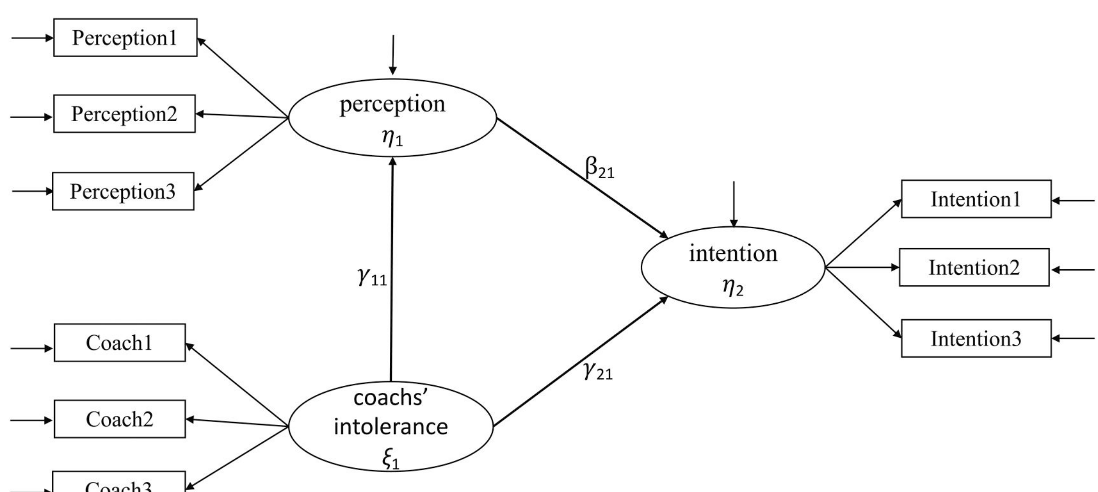

Figure 3. A mediation model for Coachs' intolerance of steroids affect players' intention via their perception (MacKinnon, 2008).

and 10.4 on the left panel, and 8.2 and 9.5 on the right panel of the table, respectively.

Dataset 3. The 3rd dataset (sample correlations and standard deviations) was originally presented by Wheaton et al. (1977), who studied stability of *alienation* as predicted by *socioeconomic status* (SES). This longitudinal dataset has been used in software manuals (e.g., Bentler, 2006; Jöreskog & Sörbom, 1993) to illustrate the SEM methodology and its applications. The dataset has $p = 6$ variables and $N = 932$ cases. The six variables are anomie 1967, powerlessness 1967, anomie 1971, powerlessness 1971, education 1966, and socioeconomic index (SEI) 1966. The path diagram in Figure 4 presents two mediation models for this dataset, where the dashed two-way arrows represent error-covariances. Let's use F41 and F42 to represent the model without and with error-covariances, respectively. For either model, the scale of ξ_1 (SES) can be phi-anchored ($\phi_{11} = 1.0$) or lambda-anchored ($\lambda_1^x = 1.0$). The model F41 has 15 parameters in $\boldsymbol{\theta}$ and 9 shared parameters in $\boldsymbol{\theta}_1$ while the model F42 has 17 parameters in $\boldsymbol{\theta}$ and 11 shared parameters in $\boldsymbol{\theta}_1$. Each model also has one indirect effect $\nu_1 = \gamma_{11}\beta_{21}$.

Fitting the model F41 (Figure 4 without error-covariances) to Dataset 3 by NML results in $T_{ml} = 71.470$, which corresponds to a p-value that is essentially zero when referred to χ^2_6. However, the comparative fit index (CFI = .969) implies that the model fits the data reasonably well (Hu & Bentler, 1999). Table A4 contains four sets of parameter estimates, their standard errors (SEs) and the corresponding z-statistics of model F41 by different parameterizations. The z-statistics/SNRs for individual parameter estimates by CORR are uniformly greater than their COV counterparts when ξ_1 is anchored by either $\phi_{11} = 1.0$ or $\lambda_1^x = 1.0$. The ratios of the z-statistics for the 9 shared parameters and the indirect effect ν_1 range from slightly above 1.0 to 1.3 on both panels of the table. The multivariate SNRs (τ^2_θ and $\tau^2_{\theta_1}$) under CORR are about 3.0 and 4.0 times of their COV counterparts on both panels of the table.

Fitting the model F42 (Figure 4 with error-covariances) to Dataset 3 results in $T_{ml} = 4.730$, corresponding to a p-value of 0.316 by $T_{ml} \sim \chi^2_4$. Results of parameter estimates, SEs and z-statistics for model F42 under different parameterizations are in Table A5. Again, the z-statistics/SNRs by CORR are uniformly larger than their COV counterparts, although their values for ψ^y_{42} are equal in the table, due to rounding. The ratios of the two sets of the z-statistics for the 11 shared parameters range from slightly above 1.0 to 1.3 on both panels of the table. The multivariate SNRs by CORR are 3.3 and 5.3 times those by COV on the left panel of Table A5, and are 3.8 and 5.4 times those by COV on the right panel of the table, respectively.

Dataset 4. Table 8.2 of Bollen (1989, p. 334) contains a sample covariance matrix of $p = 11$ industrialization-democracy variables based on a sample of $N = 75$ developing countries. As implied by Figure 5, the model has three latent constructs. *Industrialization 1960* is an exogenous construct having three indicators: gross national product (GNP) per capita, energy consumption per capita, and the percent of the labor force in industrial occupations. *Democracy 1960* is an endogenous construct having four indicators: freedom of the press, freedom of group opposition, fairness of elections, and the elective nature and effectiveness of the legislative body. *Democracy 1965* is another endogenous construct that has the same set of indicators as Democracy 1960 but measured at a later time. Bollen (1989) considered multiple models for this dataset. We will first estimate the model in Figure 5, and another model that is not scale-invariant will be examined in the later part of this section. The model in Figure 5 has $q = 31$ parameters in $\boldsymbol{\theta}$, with $q_1 = 20$ shared parameters in $\boldsymbol{\theta}_1$. There is also an indirect effect $\nu_1 = \gamma_{11}\beta_{21}$.

Fitting the model in Figure 5 to the $p = 11$ industrialization-democracy variables by NML yields $T_{ml} = 37.617$, which corresponds to p-value = .350 by $T_{ml} \sim \chi^2_{35}$. Table A6 contains the results of CORR and COV. Except for four measurements-error covariances, the z-statistics for the other 27 parameters by CORR are uniformly greater than their counterparts by COV. The four error-covariances are ψ^y_{51}, ψ^y_{42}, ψ^y_{84}, and ψ^y_{86}, and estimates for three of them are not statistically significant at the level of .05. For the 20

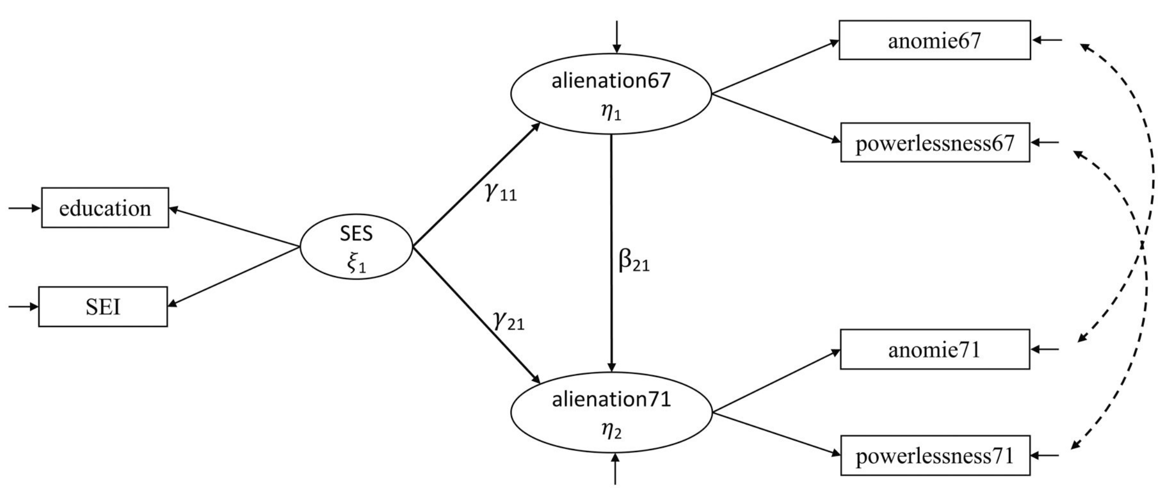

Figure 4. A model for stability of alienation with a longitudinal dataset (Bentler, 2006; Jöreskog & Sörbom, 1993; Wheaton et al., 1977).

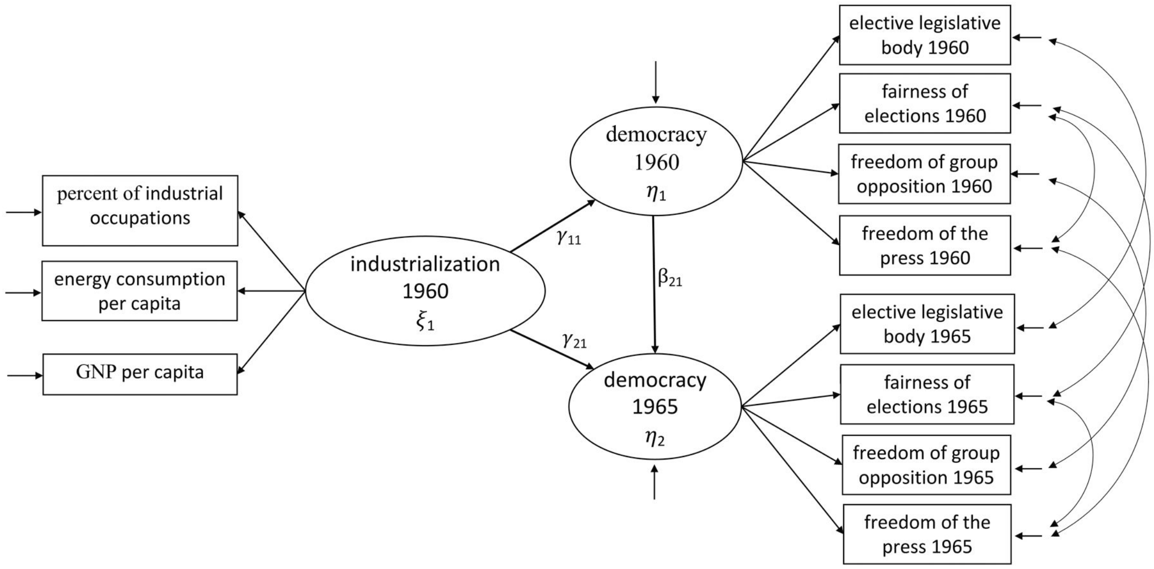

Figure 5. A model of industrialization and democracy (Bollen, 1989), each of the four factor loadings on democracy 1960 equals that on democracy 1965.

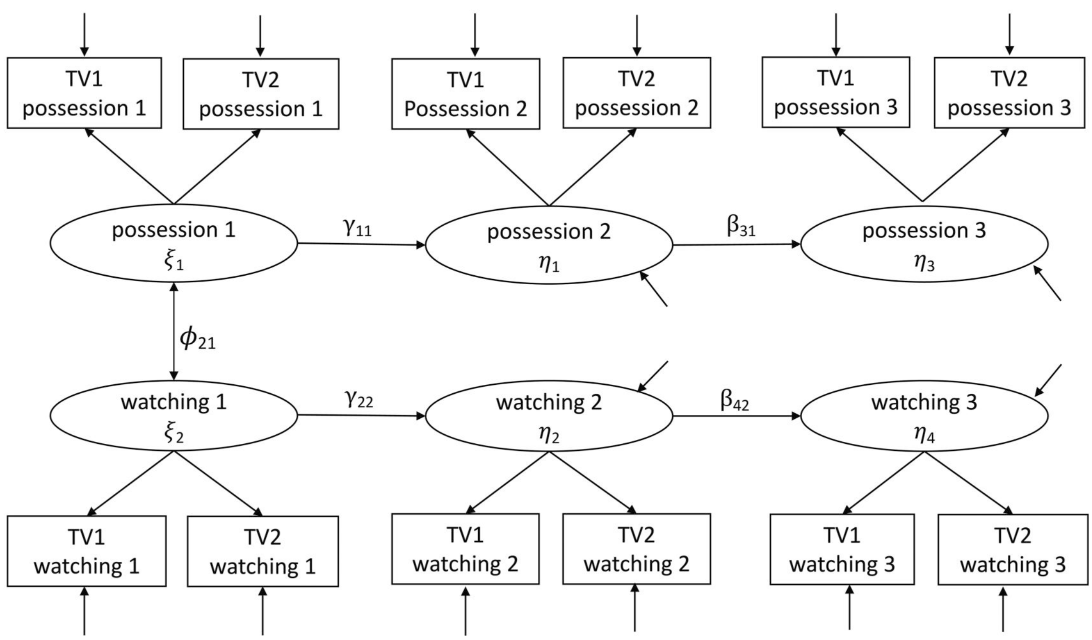

Figure 6. A longitudinal model for two versions of television possession and watching at three occasions (Jöreskog & Sörbom, 1993; Wiley & Hornik, 1973).

shared parameters (including the error covariances), the ratios of the z-statistics by CORR over those by COV range from 0.99 to 5.2 on the left panel of the table, and from .99 to 3.9 on the right panel of the table. For the multivariate SNRs, the ratios of the values of τ_θ^2 and $\tau_{\theta_1}^2$ by CORR over those by COV are 21.5 and 16.9 on the left panel of Table A6, and 21.2 and 15.9 on the right panel of the table, respectively.

3.2.4. Auto-Regressive Model

Dataset 5. The dataset (sample covariance matrix) is from Table 7.4 of the LISREL manual (Jöreskog & Sörbom, 1993, p. 193), which was from Wiley and Hornik (1973) who studied communication processes with data collected from El Salvador. The path diagram presented in Figure 6 is for a model given by Jöreskog and Sörbom (1993, p. 194), which is an auto regression model with two constructs: *television watching* by children and *television possession* by their family. Each construct was repeatedly measured at three time points. With two indicators for each construct, the sample covariance matrix for the $p = 12$ variables was based on a dataset with $N = 189$ participants. The model has 29 parameters in $\boldsymbol{\theta}$ and 17 shared parameters in $\boldsymbol{\theta}_1$. There are also two indirect effects in the structural model, which are ξ_1 on η_3 ($\nu_1 = \gamma_{11}\beta_{31}$) and ξ_2 on η_4 ($\nu_2 = \gamma_{22}\beta_{42}$).

Fitting the model in Figure 6 to Dataset 5 by NML yields $T_{ml} = 185.378$, and the p-value is essentially 0 according to $T_{ml} \sim \chi^2_{49}$. However, with CFI $=$ .938, the model can be regarded as acceptable in real data analysis. Table A7 contains the results of the model in Figure 6 by different parameterizations. There is a factor correlation ϕ_{21} when ξ_1

and ξ_2 are phi-anchored, and the z-statistics for this parameter remain the same between CORR and COV. There are two prediction-error variances (ψ_{11}^{ζ} and ψ_{33}^{ζ}) for which CORR corresponds to slightly smaller z-statistics than COV. Except for these three parameters, the z-statistics for all the other parameters by CORR are uniformly greater than their counterparts by COV. Except for ϕ_{21}, ψ_{11}^{ζ} and ψ_{33}^{ζ}, the ratios of the z-statistics by CORR over those by COV for the 17 shared parameters range from slightly above 1.0 to 5.4 on both panels of the table. The multivariate SNRs (τ_{θ}^2 and $\tau_{\theta_1}^2$) by CORR are respectively 79 and 50 times those by COV on both panels of Table A7.

3.2.5. General SEM Model

Dataset 6. Table 10.1 of Schumacker and Lomax (2010, p. 202) contains a sample covariance matrix for a study of home resource and educational achievement, which is based on a sample with $p = 9$ manifest variables and $N = 200$ participants. The model, represented by Figure 7, as was given in Figure 10.1 of Schumacker and Lomax (2010, p. 196), contains four latent constructs. According to the model, the effects of both *home resource* and *ability* on *achievement* are mediated by *aspiration*. The model has 24 parameters in $\boldsymbol{\theta}$ and 15 shared parameters in $\boldsymbol{\theta}_1$, and two indirect effects: ξ_1 on η_2 via η_1 ($\nu_1 = \gamma_{11}\beta_{21}$), and ξ_2 on η_2 via η_1 ($\nu_2 = \gamma_{12}\beta_{21}$).

Fitting the model in Figure 7 to Dataset 6 results in $T_{ml} = 57.167$, which corresponds to a p-value of .00003 according to $T_{ml} \sim \chi^2_{21}$. However, fit index CFI = .974 would suggest that the model fits the data reasonably well. Table A8 contains the results of COV and CORR for the model in Figure 7. As expected, when the scales of ξ_1 and ξ_2 are phi-anchored, the z-statistics for the factor correlation ϕ_{21} remain the same under CORR and COV. Another exception is for the parameter ψ_{22}^{ζ} for which COV corresponds to a greater z-statistic than CORR. For the other 22 parameters, the z-statistics by CORR are uniformly greater that those by COV. Except for ϕ_{21}, the ratios of the z-statistics by CORR over those by COV for the other 13 shared parameters range from slightly greater than 1.0 to 2.6 on the left panel of Table A8, and from slightly above 1.0 to 2.3 on the right panel of the table. The multivariate SNRs by CORR are 13.8 and 11.8 times those by COV on the left panel of the table, and 13.2 and 10.9 times on the right panel of the table, respectively.

Dataset 7. Table 14.1 of Kline (2016, p. 342) contains sample correlations and standard deviations of $p = 12$ variables. The data were originally from Houghton and Jinkerson (2007) who studied constructive thought strategies and job satisfaction, based on a sample of size $N = 263$ university employees. With three indicators for each latent construct, the 12 variables respectively measure four constructs: (1) *constructive thinking*, (2) *dysfunctional thinking*, (3) *subjective well-being*, and (4) *job satisfaction*. Kline (2016) considered two models for the dataset. The first model, as represented by the solid arrows in Figure 8, hypothesizes that constructive thinking reduces dysfunctional thinking, which leads to an enhanced sense of well-being, which in turn results in greater job satisfaction. Let's term this model as F81, which has 28 parameters in $\boldsymbol{\theta}$ and 16 shared parameters in $\boldsymbol{\theta}_1$. The model also has three indirect effects: $\nu_1 = \gamma_{11}\beta_{21}$ for the effect of ξ_1 on η_2 via η_1; $\nu_2 = \gamma_{11}(\beta_{21}\beta_{32} + \beta_{31})$ for the effect of ξ_1 on η_3 via η_1 and η_2; and $\nu_3 = \beta_{32}\beta_{21}$ for the effect of η_1 on η_3 via η_2. The 2nd model includes both the solid and dashed arrows, which hypothesizes that constructive thinking also has direct effects on the sense of well-being and job satisfaction. Let's term the model that includes the two dashed arrows in Figure 8 as F82, which has 30 parameters in $\boldsymbol{\theta}$ and 18 shared parameters in $\boldsymbol{\theta}_1$. The three indirect effects in F82 are: $\nu_1 = \gamma_{11}\beta_{21}$ for the effect of ξ_1 on η_2 via η_1; $\nu_2 = \gamma_{11}(\beta_{32}\beta_{21} + \beta_{31}) + \gamma_{21}\beta_{32}$ for the effect of ξ_1 on η_3 via η_1 and η_2; and $\nu_3 = \beta_{32}\beta_{21}$ for the effect of η_1 on η_3 via η_2.

Fitting the model F81 to Dataset 7 yields $T_{ml} = 66.061$, corresponding to a p-value of .064 by $T_{ml} \sim \chi^2_{50}$. With RMSEA = .035 and CFI = .984, the model would be regarded as fitting the data adequately in practice (Hu & Bentler, 1999). Table A9 contains the results for model F81. The z-statistics for all the direct and indirect effects (28 free parameters and 3 indirect effects) by CORR are uniformly greater than their counterparts by COV. The ratios between the two sets of z-statistics or SNRs for the 16 shared parameters range from slightly above 1.0 to 1.8 on both the left and right panels of Table A9. For the model F81, the

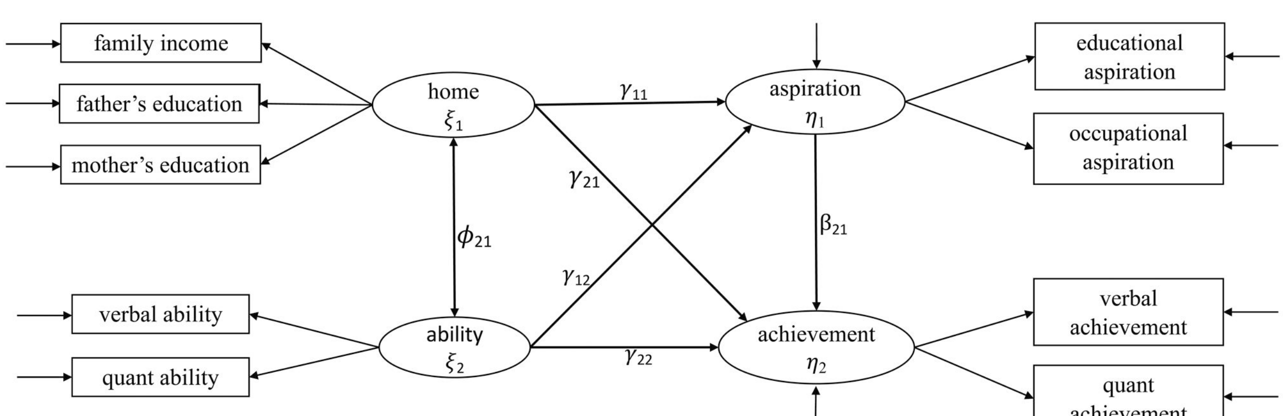

Figure 7. A mediation model for ability and home on educational achievement (Schumacker & Lomax, 2010).

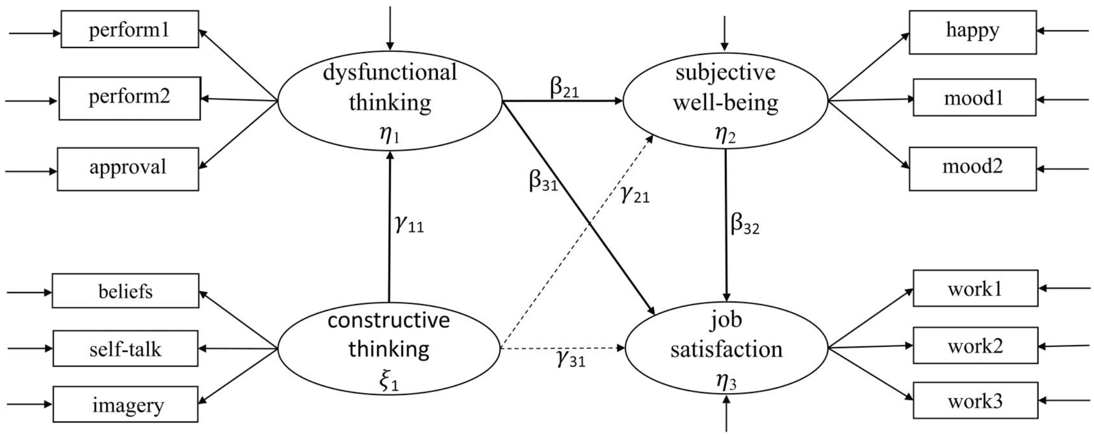

Figure 8. Two mediated models for thought strategies on job satisfaction (Houghton & Jinkerson, 2007; Kline, 2016).

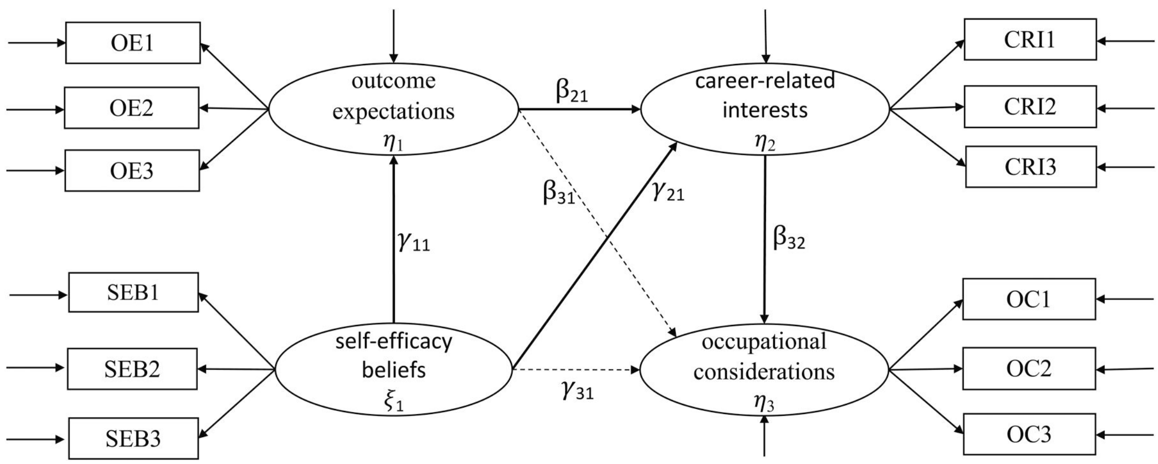

Figure 9. Two mediated models for self-efficacy belief on occupational considerations (Weston & Gore, 2006).

multivariate SNRs (τ_θ^2 and $\tau_{\theta_1}^2$) by CORR are 2.8 and 4.4 times those by COV on the left panel, and 2.6 and 4.2 times those by COV on the right panel, respectively. There exist a few estimates that are not statistically significant at the level of .05.

Fitting the model F82 to Dataset 7 results in $T_{ml} = 62.231$, corresponding to a p-value = .081 by $T_{ml} \sim \chi^2_{48}$. RMSEA = .034 and CFI = .986 also imply that the model fits the data well. Table A10 contains the results by COV and CORR for estimating the model F82, together with three indirect effects. The z-statistics for both direct and indirect effects by CORR are uniformly greater than their counterparts by COV. The ratios between the two sets of z-statistics for the 18 shared parameters range from slightly above 1.0 to 1.8 on both the left and right panels of Table A10. The values of τ_θ^2 and $\tau_{\theta_1}^2$ by CORR are 2.8 and 4.4 times those by COV on the left panel, and 2.6 and 4.3 times on the right panel of the table. Table A10 also contains a few estimates that are not statistically significant at the level of .05.

Dataset 8. In a tutorial article on SEM, Weston and Gore (2006, Table 2) presented a sample covariance matrix for a dataset with $N = 403$ cases and $p = 12$ variables. The data were from a survey of college students who participated in a vocational psychology research project. With three indicators for each construct, the 12 variables respectively measure (1) *self-efficacy beliefs*, (2) *outcome expectations*, (3) *career-related interests*, and (4) *occupational considerations*. Weston and Gore considered two structural models, as represented by Figure 9. The model by the solid arrows (model F91) implies that the effect of self-efficacy beliefs on career-related interests is partially mediated by outcome expectations, while the effect of self-efficacy beliefs on occupational considerations is completely mediated through outcome expectations and career-related interests. Model F91 has 28 parameters in $\boldsymbol{\theta}$, 16 shared parameters in $\boldsymbol{\theta}_1$, and three indirect effects: $\nu_1 = \beta_{21}\gamma_{11}$ for the effect of ξ_1 on η_2 via η_1; $\nu_2 = \beta_{32}\beta_{21}\gamma_{11} + \beta_{32}\gamma_{21}$ for the effect of ξ_1 on η_3 via η_1 and η_2; and $\nu_3 = \beta_{32}\beta_{21}$ for the effect of η_1 on η_3 via η_2.

Model F92, represented by both the solid and dashed arrows in Figure 9, implies that the effects of self-efficacy beliefs on career-related interests and on occupational considerations are partially mediated by outcome expectations; and the effect of outcome expectations on occupational considerations is also partially mediated by career-related interests. Model F92 has 30 parameters in $\boldsymbol{\theta}$, 18 shared parameters in $\boldsymbol{\theta}_1$, and three indirect effects: $\nu_1 = \beta_{21}\gamma_{11}$ for the effect of ξ_1 on η_2 via η_1; $\nu_2 = (\beta_{32}\beta_{21} + \beta_{31})\gamma_{11} + \beta_{32}\gamma_{21}$ for the effect of ξ_1 on η_3 via η_1 and η_2; and $\nu_3 = \beta_{32}\beta_{21}$ for the effect of η_1 on η_3 via η_2.

Fitting the model F91 to Dataset 8 results in $T_{ml} = 416.061$, which corresponds to a p-value that is essentially 0 according to $T_{ml} \sim \chi^2_{50}$. With CFI = .913, and RMSEA = .135, the model might not be regarded as fitting the data adequately although it is substantively sound (see Weston & Gore, 2006 and references therein). Table A11 presents the results for model F91, where the z-statistics for both direct and indirect effects by CORR are uniformly greater than their counterparts by COV on both the left and right panels of the table. The ratios between the two sets of z-statistics for the 16 shared parameters range from slightly above 1.0 to 3.1 on the left panel of the table, and from slightly above 1.0 to 2.6 on the right panel of the table. The multivariate SNRs τ^2_θ and $\tau^2_{\theta_1}$ by CORR are 27.9 and 20.2 times those by COV on the left panel, and 30.5 and 20.7 times those by COV on the right panel of Table A11, respectively.

Estimating the model F92 results in $T_{ml} = 361.848$, with $df = 48$, which corresponds to RMSEA = .128 and CFI = .926. The partial mediation model F92 might be regarded as fitting the data reasonably well in practice although the values of the fit indices are not up to the established standard (Hu & Bentler, 1999). Table A12 contains the results for parameters of model F92, where the z-statistics by CORR on both the left and right panels are uniformly greater than their counterparts by COV. The ratios of the two sets of z-statistics for the 18 shared parameters range from slightly above 1.0 to 3.0 on the left panel, and from slightly above 1.0 to 2.6 on the right panel of the table. The ratios of τ^2_θ and $\tau^2_{\theta_1}$ by CORR to those by COV are 27.6 and 20.2 on the left panel, and 29.6 and 20.7 on the right panel of Table A12, respectively.

Dataset 9. This is a raw dataset from Neumann (1994), who studied the relationship of psychopathology and alcoholism. The dataset consists of $p = 10$ variables and $N = 335$ cases, and the 10 variables were designed to measure 5 constructs. The only exogenous construct (*family history*) is tapped by two indicators: family history of psychopathology and family history of alcoholism. The other eight indicators are respectively the age of 1st problem with alcohol, age of 1st detoxification from alcohol, alcohol severity score, alcohol use inventory, SCL-90 psychological inventory, the sum of the Minnesota Multiphasic Personality Inventory scores, the lowest level of psychosocial functioning during the past year, and the highest level of psychosocial functioning during the past year. With two indicators for each latent construct, these eight indicators respectively measure: *age of onset*, *alcohol symptoms*, *psychopathology symptoms*, and *global functioning*. Neumann's (1994) theoretical model for this data set is represented by Figure 10, which has 26 parameters in $\boldsymbol{\theta}$ and 16 shared parameters in $\boldsymbol{\theta}_1$. The model also has six indirect effects, and they are respectively given by $\nu_1 = \beta_{21}\gamma_{11}$, ξ_1 on η_2; $\nu_2 = \gamma_{11}(\beta_{31} + \beta_{32}\beta_{21})$, ξ_1 on η_3; $\nu_3 = \gamma_{11}\beta_{43}(\beta_{31} + \beta_{32}\beta_{21}) + \beta_{42}\beta_{21}\gamma_{11}$, ξ_1 on η_4; $\nu_4 = \beta_{32}\beta_{21}$, η_1 on η_3; $\nu_5 = \beta_{43}(\beta_{31} + \beta_{32}\beta_{21}) + \beta_{21}\beta_{42}$, η_1 on η_4; and $\nu_6 = \beta_{43}\beta_{32}$, η_2 on η_4.

With individual observations, we can check the distribution properties of this psychopathology-alcoholism sample, which has a standardized multivariate kurtosis $M_s = 14.763$ (Mardia, 1970). Consequently, we applied a robust transformation to the raw data. By adjusting a tuning parameter in the transformation, we end up with $M_s = -.110$ (see Yuan et al., 2000). We regard the transformed sample as approximately normally distributed. In addition, because the variables have very different standard deviations, ranging from 1.5 to 55.7, we divided the variables respectively by 10.0, 55.0, 16.0, 14.0, 1.0, 2.0, 11.0, 10.0, 8.0, 5.0 so that their resulting SDs are between 1 and 2.

Fitting the model in Figure 10 to the robustly transformed data by NML results in $T_{ml} = 40.985$, corresponding to a p-value = .069 when referred to χ^2_{29}. RMSEA = .035

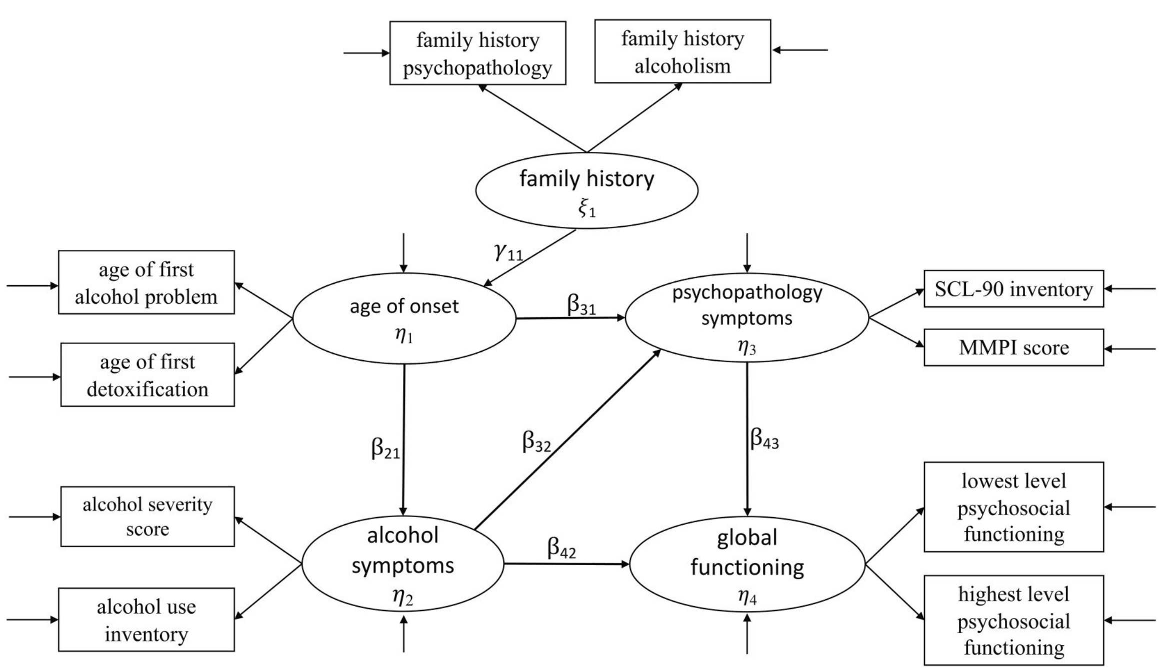

Figure 10. A mediation model of symptoms of alcoholism on psychopathology with a longitudinal dataset (Neumann, 1994).

and CFI = .987 also indicate that the model fits the data very well. Table A13 contains the results for estimating the model parameters by COV and CORR together with the indirect effects. The z-statistics for all the parameters by CORR are uniformly greater than their counterparts by COV. The ratio of the SNRs by CORR over those by COV for the 16 shared parameters range from slightly above 1.0 to 1.3 on both left and right panels of the table. The multivariate SNRs (τ_θ^2 and $\tau_{\theta_1}^2$) by CORR are 2.6 and 4.2 times their COV counterparts on the left panel of Table A13, and 2.5 and 4.2 times those by COV on the right panel of the table.

Dataset 10. This is also a raw dataset from a study of health and stress, and was examined in Yuan and Deng (2021), with $p = 24$ and $N = 264$. The 264 participants were recruited from both high-pressure professionals and from a psychiatric hospital. The 24 variables are indicators of four constructs. The three exogenous constructs are respectively: (1) *emotional exhaustion* with 5 indicators, (2) *cynicism* with 4 indicators, and (3) *professional efficacy* with 6 indicators. The only endogenous construct (*depression*) has 9 indicators obtained from patient health questionnaire (PHQ9, Kroenke, Spitzer & Williams, 2001). The path diagram for the structural model is in Figure 11, which has 30 shared parameters in $\boldsymbol{\theta}_1$ and 54 parameters in $\boldsymbol{\theta}$.

With a standardized multivariate kurtosis $M_s = 23.292$ for the observed data, we first applied a robust transformation to the sample such that the transformed data has $M_s = -.003$ (see Yuan & Deng, 2021), and our following results are obtained by applying the NML method to the robustly-transformed data.

Fitting the model in Figure 11 to Dataset 10 results in $T_{ml} = 469.579$, which has a p-value that is essentially 0 according to $T_{ml} \sim \chi^2_{246}$. However, fit indices CFI = .948 and RMSEA = .059 suggest that the model fits the data reasonably well. Table A14 contains the results of parameter estimates of the model. When the scales of the three exogenous latent variables are anchored by $\phi = 1.0$ in the left panel of the table, the z-statistics for the three factor correlations remain the same between the two parameterizations. The z-statistics for the other parameters by CORR are uniformly greater than their counterparts by COV. For the 30 shared parameters, the ratios of the z-statistics by CORR over those by COV range from slightly above 1.0 to 2.8 on the left panel, and from slightly above 1.0 to 2.5 on the right panel of Table A14. The values of the multivariate SNRs τ_θ^2 and $\tau_{\theta_1}^2$ by CORR are 5.0 and 7.7 times those by COV on the left panel, and 6.8 and 7.1 times those by COV on the right panel of the table, respectively.

3.2.6. Non-Recursive Model

The 14 models we have studied so far do not have any reciprocal relationships, also termed as non-recursive models. We next present two examples that contain such relationships.

Dataset 11. This dataset is a correlation matrix used by Loehlin and Beaujean (2017, p. 98) to illustrate the utility of SEM in modeling reciprocal causal relationships. The data were originally from Maruyama and McGarvey (1980) who studied the effect of school desegregation. With a sample of size $N = 249$, the dataset consists of 13 indicators for 5 latent variables. They are *acceptance by adults* (ASA), measured by father's evaluation (FEV), mother's evaluation (MEV), teacher's evaluation (TEV); *socioeconomic status* (SES), measured by socioeconomic index (SEI), education of

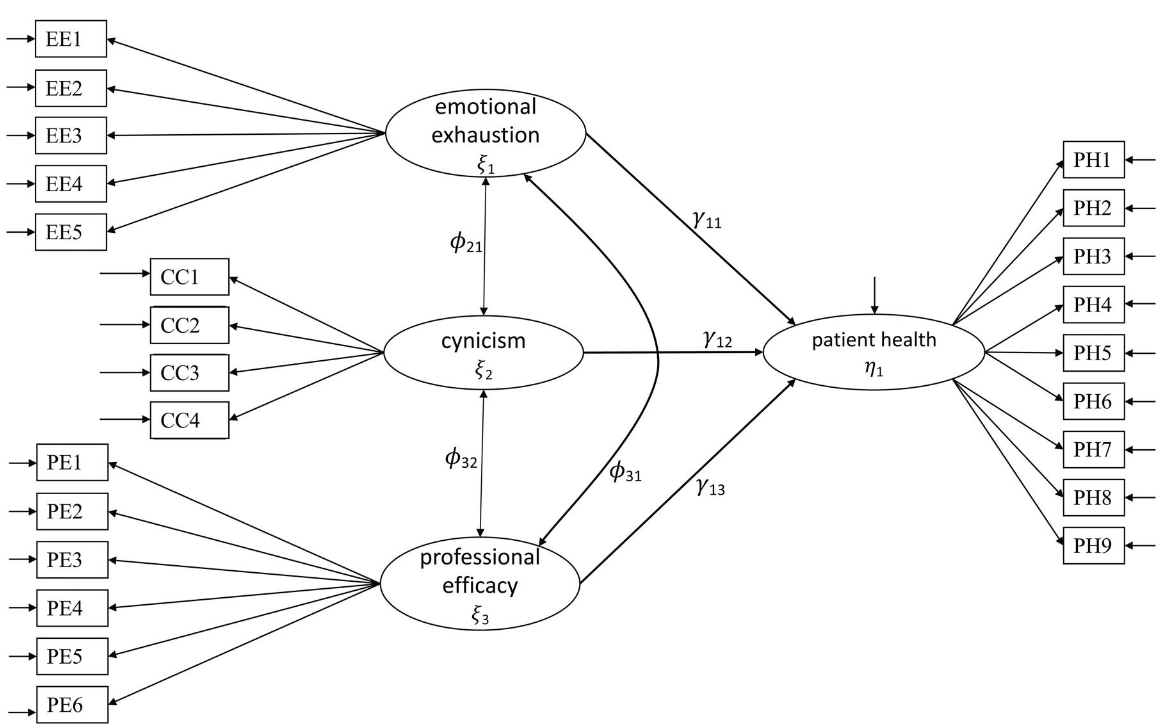

Figure 11. A latent-variable model of burnout and depression (Yuan & Deng, 2021).

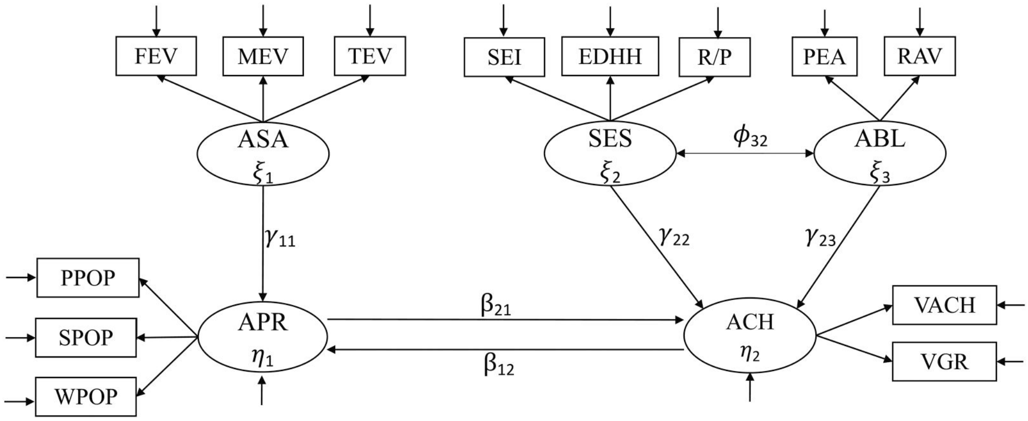

Figure 12. A non-recursive model between acceptance by peers and achievement (Loehlin & Beaujean, 2017; Maruyama & McGarvey, 1980).

head of household (EDHH), rooms per person in house (R/P); *ability* (ABL), measured by Peabody picture vocabulary test (PEA), Raven's progressive matrices test (RAV); *acceptance by peers* (APR), measured by playground popularity (PPOP), seating popularity (SPOP), schoolwork popularity (WPOP); and *academic achievement* (ACH), measured by verbal achievement score (VACH), and verbal grades (VGR). The model represented by the path diagram in Figure 12 was proposed by Maruyama and McGarvey (1980), which has 19 shared parameters in $\boldsymbol{\theta}_1$ and 32 parameters in $\boldsymbol{\theta}$. Note that, with reciprocal relationships, η_1 and η_2 have indirect effects on each other and on themselves. There are a total of 10 indirect effects in the structural model, represented by ν_1 to ν_{10} for the effects of ξ_1 on η_1, ξ_2 on η_1, ξ_3 on η_1, ξ_1 on η_2, ξ_2 on η_2, ξ_3 on η_2, η_1 on η_1, η_2 on η_1, η_1 on η_2, η_2 on η_2, respectively. Because the expressions of these effects are tedious and are not the focus of the current article, they are not presented here. Interested readers are referred to Bollen (1989, pp. 376–382) and Bentler and Freeman (1983) for direct and indirect effects with non-recursive models.

Fitting the model in Figure 12 to Dataset 11 results[5] in $T_{ml} = 138.436$, which corresponds to a p-value that is essentially 0 when being referred to χ^2_{59}. However, RMSEA = .074 may suggest that the model fits the data reasonably well. Table A15 contains the results of parameter estimates of the model by COV and CORR. Note that ξ_1 is uncorrelated with ξ_2 and ξ_3 in Figure 12. Also note that the indirect effect of η_1 on η_1 via η_2 is identical to that of η_2 on η_2 via η_1. When the three exogenous latent variables are anchored by $\phi = 1.0$, except for the factor correlation ϕ_{32}, the z-statistics of the estimates for all the other parameters by CORR are uniformly greater than their counterparts by COV. Due to rounding, the two parameterizations have identical values of z-statistics for three of the ten indirect effects in Table A15. The ratios of the two sets of z-statistics for the 19 shared parameters range from slightly above 1.0 to 1.3 on both the left and right panels of the table. For the multivariate SNR, the values of τ^2_{θ} and $\tau^2_{\theta_1}$ by CORR over those by COV are 1.6 and 2.6 on the left panel, and 1.7 and 2.5 on the right panel of Table A15, respectively.

Dataset 12. The 12th dataset was originally from Duncan, Haller and Portes (1968), who studied reciprocal relationships between respondent's ambition and best friend's ambition. The dataset was used by Jöreskog and Sörbom (1993, section 1.5.2) as an example for using the LISREL program to estimate non-recursive SEM models. The same dataset was also used by Loehlin and Beaujean (2017, pp. 115–120) to illustrate issues in modeling reciprocal relationships with correlated errors. The dataset consists of $p = 10$ variables from $N = 329$ participants. They are x_1=respondent's parental aspiration, x_2=respondent's intelligence, x_3=respondent's socioeconomic status, x_4=best friend's socioeconomic status, x_5=best friend's intelligence, x_6=best friend's parental aspiration, y_1=respondent's occupational aspiration, y_2=respondent's educational aspiration, y_3=best friend's educational aspiration, y_4=best friend's occupational aspiration. As implied by Figure 13, x_1 to x_6 are freely correlated and serve as covariates of the two latent variables: η_1 = *respondent's ambition* and η_2 = *best friend's ambition*, which are measured by y_1 and y_2, and y_3 and y_4, respectively. The model has 39 parameters in $\boldsymbol{\theta}$ and 29 parameters in $\boldsymbol{\theta}_1$. There are 16 indirect effects in the model represented by ν_1 to ν_{16}, which are respectively the effects of x_1 to x_6 on η_1, x_1 to x_6 on η_2, η_1 on η_1, η_2 on η_1, η_1 on η_2, and η_2 on η_2.

Fitting the model in Figure 13 to Dataset 12 yields $T_{ml} = 26.893$, which corresponds to p-value = .043 by $T_{ml} \sim \chi^2_{16}$. RMSEA = .046 and CFI = .987 suggest that the model fits the data well. Table A16 contains the results by COV and CORR. Note that, without exogenous latent variables in the model, there is only one pair of results to compare. For the 39 parameters in $\boldsymbol{\theta}$, the z-statistics by CORR are uniformly larger than their counterparts by COV in Table A16. Except for the indirect effects η_1 on η_1 (ν_{13}) and η_2 on η_2 (ν_{16}) that CORR and COV have equal z-statistics due to rounding, the z-statistics for the other 14 indirect effects by CORR are also uniformly greater than those by COV. The ratios of the two sets of z-statistics for the 29 shared parameters range from slightly above 1.0 to 1.3. The multivariate SNRs by CORR over those by COV are 2.4 and 3.5, respectively.

[5]Loehlin and Beaujean (2017, p. 99) reported the value of T_{ml} as 140.30, and our value of T_{ml} is slightly smaller. This might be because we used the correlation matrix of Maruyama and McGarvey (1980), which has 3 decimals.

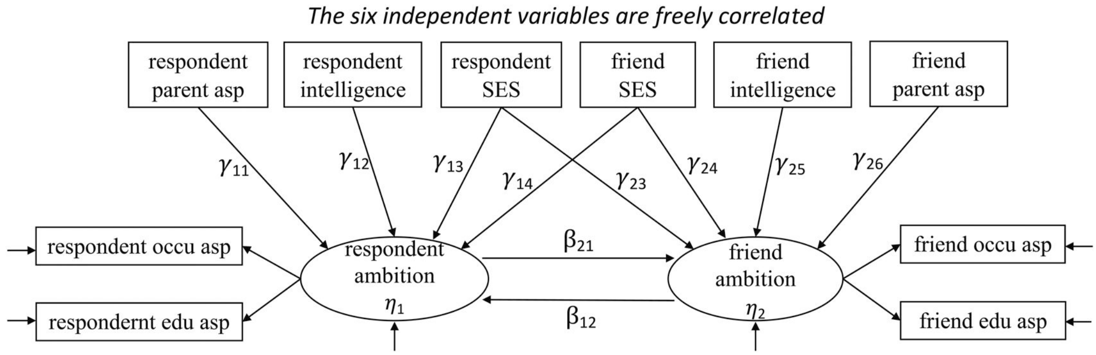

Figure 13. A non-recursive model with reciprocal causation between two latent variahles (Jöreskog & Sörbom, 1993; Duncan, Haller & Portes 1968).

Several estimates in Table A16 are not statistically significant at the level of .05.

3.2.7. Not-Scale-Invariant Model

All the 16 models we have examined thus far are scale-invariant, simply because none of them have constraints beyond the needs for scaling the latent variables. We next consider two models that are not scale-invariant.

For Dataset 4 (industrialization-democracy data) and the path diagram in Figure 5, Bollen (1989) contains a model for the four loadings on η_1 to equal those on η_2. That is, $\lambda_1^y = \lambda_5^y = 1.0$, $\lambda_2^y = \lambda_6^y$, $\lambda_3^y = \lambda_7^y$, $\lambda_4^y = \lambda_8^y$. Because of the three substantive constraints, the resulting model is no longer scale-invariant. Nevertheless, each parameterization has its own theoretical implication, as discussed in Section 2.3. Fitting this model to Dataset 4 results in $T_{ml} = 39.644$ by COV and $T_{ml*} = 41.624$ by CORR, which correspond to p-values .397 and .316 when the statistics are referred to χ^2_{38}, respectively. Thus, both parameterizations/models fit the data well.

Table A17 contains the results of four sets of parameter estimates. There are four measurements-error covariances (ψ^y_{42}, ψ^y_{51}, ψ^y_{73}, ψ^y_{84}) and one prediction-error variance (ψ^ζ_{22}) that COV yields slightly greater z-statistics than CORR on both the left and right panels of Table A17. For the estimates of the other 23 parameters, the z-statistics by CORR are uniformly greater than their COV counterparts. Note that there are 17 free parameters shared by the two parameterizations. Except for the four measurement-error covariances and one prediction-error variance, the ratios of the two sets z-statistics for the other 12 shared parameters range from slightly above 1.0 to 5.1 on the left panel of the table, and from slightly above 1.0 to 3.9 on the right panel of the table. The values of τ^2_θ and $\tau^2_{\theta_1}$ by CORR are 20.6 and 16.4 times those by COV on the left panel, and 29.6 and 20.7 times those by COV on the right panel of Table A17, respectively.

For Dataset 12 and the non-recursive model in Figure 13, Jöreskog and Sörbom (1993, p. 43) also estimated a model with $\beta_{21} = \beta_{12}$, implying that a respondent and his/her best friend equally affect each other with respect to ambition. This additional constraint makes the model not scale-invariant. However, the constraint represents an important substantive reasoning. Fitting the model in Figure 13 with $\beta_{21} = \beta_{12}$ to Dataset 12 by COV results in $T_{ml} = 26.961$, corresponding to a p-value = .059 according to $T_{ml} \sim \chi^2_{17}$. Estimating the model with $\beta_{21} = \beta_{12}$ under CORR results in $T_{ml*} = 26.899$, corresponding to a p-value = .060. Thus, the two parameterizations fit the data about equally well although the model is not scale-invariant. Table A18 contains the results by COV and CORR, and we only have one pair of estimates for each parameter because the model does not have exogenous latent variables. There are 38 parameters in $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_*$, and 16 parameters for indirect effect. With a total of 54 parameters, COV delivers greater z-statistics than CORR only for γ_{23} and ϕ_{41}. For the other 26 shared parameters, the ratios of the z-statistics by CORR over those by COV range from slightly above 1.0 to 1.35. The values of the multivariate SNRs τ^2_θ and $\tau^2_{\theta_1}$ by CORR are 2.4 and 3.5 times those by COV, respectively.

We have presented results for comparing LISREL-correlation against LISREL-covariance with 18 models and 12 datasets. Among them, 10 datasets (sample covariance and correlation matrices) and 16 models are from SEM textbooks, software manuals, and tutorial articles. The goodness of fit by the 18 models ranges from marginally to closely. But all the models were previously proposed in the cited references rather than newly created. Across the 18 models, LISREL-correlation not only yields consistent SEs for parameter estimates but also more efficient/accurate estimates than the conventional LISREL-covariance models.

To summarize the results of the two parameterizations, Table 1 contains the values of the multivariate SNR ($\hat{\tau}^2_{\theta_1}$ & $\hat{\tau}^2_{\theta_{*1}}$) for shared parameters as well as the ratio $\hat{\kappa} = \hat{\tau}^2_{\theta_{*1}}/\hat{\tau}^2_{\theta_1}$ across the 18 models with the 12 datasets. When the exogenous latent variables are anchored by $\phi = \mathrm{Var}(\xi) = 1.0$, the value of $\hat{\kappa}$ ranges from 2.6 to 53.8, implying that the efficiency of $\hat{\boldsymbol{\theta}}_{*1}$ is 2.6 to 53.8 times that of the corresponding $\hat{\boldsymbol{\theta}}_1$. When the exogenous latent variables are lambda-anchored by fixing its first loading at 1.0, the value of $\hat{\kappa}$ ranges from 2.5 to 50.7, implying that the efficiency of $\hat{\boldsymbol{\theta}}_{*1}$ is 2.5 to 50.7 times that of the corresponding $\hat{\boldsymbol{\theta}}_1$. The values of the two SNRs are inversely proportional to the needed sample sizes for the two parameterizations to achieve equal accuracy of parameter estimates or statistical power for testing H_0: $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_{*1} = \mathbf{0}$, although specific interest for this type of hypothesis may need to be justified in a substantive context. Take the mediation model in Figure 4 (without correlated errors) for example. With $\hat{\tau}^2_{*1} = 5.897$

Table 1. Multivariate signal-to-noise ratio (SNR) for shared parameters under LISREL-correlation (CORR) and LISREL-covariance (COV) as well as the ratio $\hat{\kappa} = \hat{\tau}^2_{\theta_{*1}}(\text{CORR})/\hat{\tau}^2_{\theta_1}(\text{COV})$ with 12 datasets and 18 models.

Model feature					Data			Multivariate SNR					
								ϕ-anchored ξ			λ-anchored ξ		
	p	m	q	q_1	set	Fig	TabA	$\hat{\tau}^2_{\theta_1}$	$\hat{\tau}^2_{\theta_{*1}}$	$\hat{\kappa}$	$\hat{\tau}^2_{\theta_1}$	$\hat{\tau}^2_{\theta_{*1}}$	$\hat{\kappa}$
confirmatory factor	9	3	21	12	1	1	1	2.984	16.136	5.4	6.073	24.740	4.1
hierarchical factor	9	4	21	12	1	2	2	7.533	31.899	4.2	7.910	37.589	4.8
mediation	9	3	21	12	2	3	3	8.362	87.105	10.4	9.774	92.918	9.5
mediation	6	3	15	9	3	4	4	5.094	22.502	4.4	5.897	26.556	4.5
mediation (e cov)	6	3	17	11	3	4	5	5.032	26.870	5.3	5.873	31.942	5.4
mediation (e cov)	11	3	31	20	4	5	6	12.612	213.215	16.9	17.663	280.006	15.9
auto regression	12	6	29	17	5	6	7	22.048	1186.337	53.8	32.226	1633.121	50.7
general SEM	10	4	24	15	6	7	8	8.216	97.066	11.8	13.226	143.533	10.9
general SEM (c med)	12	4	28	16	7	8	9	8.499	37.130	4.4	9.026	38.293	4.2
general SEM (p med)	12	4	30	18	7	8	10	8.510	37.374	4.4	9.087	38.794	4.3
general SEM (c med)	12	4	28	16	8	9	11	16.899	341.613	20.2	20.360	421.826	20.7
general SEM (p med)	12	4	30	18	8	9	12	15.221	307.432	20.2	18.685	386.845	20.7
general SEM	10	5	26	16	9	10	13	6.720	28.360	4.2	7.059	29.935	4.2
general SEM	24	4	54	30	10	11	14	8.877	68.259	7.7	18.006	128.640	7.1
non-recursive	13	5	32	19	11	12	15	5.749	15.180	2.6	7.131	18.149	2.5
non-recursive	10	2	39	29	12	13	16				6.608	23.339	3.5
not scale-invariant	11	3	28	17	4	5	17	12.445	203.989	16.4	17.500	269.299	15.4
not scale-invariant	10	2	38	16	12	13	18				6.605	23.339	3.5

Note. (e cov) = the model contains error covariance, (c med) = the model implies complete mediation, (p med) = the model implies paritial mediation, p = # of manifest variables, m = # of latent variables, q =# of parameters in $\boldsymbol{\theta}$, q_1 =# of parameters in $\boldsymbol{\theta}_1$, Fig = Figure, TabA = Table A.

and $\hat{\tau}_1^2 = 26.556$ in Table A4 when the three latent variables in the figure are all anchored by fixing its first loading at 1.0, we have $\hat{\kappa} = 26.556/5.897 \approx 4.5$. This implies that a sample with $N = 50$ under LISREL-correlation would yield estimates $\hat{\boldsymbol{\theta}}_{*1}$ that are about same accurate as $\hat{\boldsymbol{\theta}}_1$ for a sample with size $N = 225$ under LISREL-covariance.

4. Monte Carlo Results

We have compared the efficiency of parameter estimates under CORR and COV across the analyses of many real datasets with different types of models. In this section, we further compare the two parameterizations via a small-scale Monte Carlo study. The advantage of a Monte Carlo study is that we have a correct model to work with and also have the control of the population values of the parameters, although the two parameterizations of the LISREL model as well as the concept SNR equally apply to imperfect models.

To make the study concrete, we consider a mediation model with three latent variables ξ_1, η_1 and η_2, and each is measured by three unidimensional indicators, as represented by Figure 3. In the population, the values of the three loadings for ξ are 1.00, 1.10, and 1.20; the values of the three loadings for η_1 are 1.20, 1.10, 1.30; and those for η_2 are .90, 1.25, 1.10, respectively. As implied by Figure 3, the structural mediation model can be written as

$$\eta_1 = \gamma_{11}\xi + \zeta_1, \quad \eta_2 = \gamma_{21}\xi + \beta_{21}\eta_1 + \zeta_2.$$

The population values of the structural parameters are $\phi_{11} = 1.00$, $\gamma_{11} = .80$, $\gamma_{21} = .60$; $\beta_{21} = .70$, $\psi_{11}^{\zeta} = .80$, $\psi_{22}^{\zeta} = .90$. The population values for the 9 measurement-error variances are set at 1.0. Let the 9×9 covariance matrix corresponding to the above parameter values be Σ_0. Our data for the study are simulated from the normal distribution $N_9(\mathbf{0}, \Sigma_0)$.

The model to fit the data is the same as the one generating our data. But the scale of each latent variable needs to be fixed in order for the model to be identified, and we let $\phi_{11} = 1.0$, $\lambda_1^y = \lambda_4^y = 1.0$ for the purpose. Although the resulting model-implied covariance matrix is the same as the one that generated the data, some parameters have changed their population values due to scaling the latent variables. In particular, under COV, the population values of the loadings for η_1 become 1.0, 0.917, 1.083; and those for η_2 become 1.0, 1.389, 1.222, respectively; and the population values of the parameters in the structural model become $\gamma_{11} = 0.960$, $\gamma_{21} = 0.540$, $\beta_{21} = 0.525$, and $\psi_{11}^{\zeta} = 1.152$, and $\psi_{22}^{\zeta} = 0.729$. While the population values for the three loadings of ξ and the 9 error variances under COV remain the same as specified for generating Σ_0. Under CORR, the population values of the parameters in $\boldsymbol{\theta}_*$ become $\lambda_1^x = 0.707$, $\lambda_2^x = 0.740$, $\lambda_3^x = 0.768$, $\lambda_1^y = 1.0$, $\lambda_2^y = 0.970$, $\lambda_3^y = 1.025$, $\lambda_4^y = 1.0$, $\lambda_5^y = 1.087$, $\lambda_6^y = 1.057$, $\phi_{11} = 1.00$, $\gamma_{11} = .548$, $\gamma_{21} = .305$; $\beta_{21} = .520$, $\psi_{11}^{\zeta} = .375$, $\psi_{22}^{\zeta} = .232$. The values of the standard deviations of the 9 manifest variables are 1.414, 1.487, 1.562, 1.753, 1.656, 1.853, 1.771, 2.263, 2.047, respectively. Note that the model has 21 free parameters in $\boldsymbol{\theta}$ with 12 shared parameters in $\boldsymbol{\theta}_1$. There is also an indirect effect $\nu_1 = \gamma_{11}\beta_{21}$. In the following, we will focus on the values of the SNRs for the parameter estimates instead of the parameter estimates themselves. In particular, the sizes of the SNRs do not depend on the specific values of ϕ_{11}, λ_1^y and λ_4^y once the anchors are determined.

We choose three conditions for sample size N: 200, 500 and 1000. For a sample of size N from $N_9(\mathbf{0}, \Sigma_0)$, the NML estimates of both $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_*$ are obtained, and so are the corresponding SNRs for both $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\theta}}_*$, parallel to those computed for the real data analyses in the previous section. The values of the indirect effect $\hat{\nu}_1 = \hat{\gamma}_{11}\hat{\beta}_{21}$ and its SNR are also obtained, and so are the multivariate SNR $\hat{\tau}_1^2$ for the 12 shared parameters (see Eq. 12). With $N_r = 1000$

Table 2. Average (ave) and empirical (emp) signal-to-noise ratios for a mediation model with 9 manifest variables and three latent variables ($p_x = 3$, $p_y = 6$, $\phi_{11} = 1$, $\lambda_1^y = \lambda_4^y = 1.0$).

	$N = 200$				$N = 500$				$N = 1000$			
θ	ave_s	emp_s	ave_r	emp_r	ave_s	emp_s	ave_r	emp_r	ave_s	emp_s	ave_r	emp_r
λ_1^x	0.742	0.722	1.120	1.069	0.741	0.744	1.108	1.111	0.741	0.739	1.106	1.111
λ_2^x	0.785	0.794	1.237	1.211	0.786	0.772	1.229	1.202	0.785	0.765	1.223	1.213
λ_3^x	0.822	0.827	1.344	1.278	0.822	0.830	1.331	1.289	0.822	0.840	1.329	1.281
λ_2^y	0.862	0.861	1.371	1.395	0.864	0.878	1.374	1.399	0.863	0.863	1.373	1.405
λ_3^y	0.916	0.932	1.448	1.453	0.918	0.919	1.451	1.460	0.918	0.929	1.451	1.490
λ_5^y	1.084	1.090	1.899	1.873	1.079	1.062	1.882	1.846	1.079	1.020	1.883	1.808
λ_6^y	1.049	1.014	1.884	1.803	1.044	1.069	1.870	1.835	1.044	1.061	1.870	1.849
γ_{11}	0.572	0.587	0.741	0.734	0.573	0.616	0.740	0.776	0.575	0.586	0.742	0.749
γ_{21}	0.286	0.283	0.300	0.293	0.287	0.281	0.300	0.291	0.288	0.286	0.301	0.296
β_{21}	0.399	0.393	0.440	0.429	0.399	0.389	0.438	0.427	0.398	0.398	0.435	0.436
ψ_{11}^ζ	0.395	0.391	0.486	0.492	0.396	0.386	0.486	0.491	0.396	0.400	0.485	0.498
ψ_{22}^ζ	0.388	0.384	0.440	0.443	0.390	0.382	0.443	0.433	0.390	0.387	0.443	0.443
ψ_{11}^x	0.548	0.523	1.414	1.364	0.549	0.533	1.414	1.369	0.549	0.544	1.414	1.382
ψ_{22}^x	0.514	0.498	1.414	1.411	0.514	0.512	1.414	1.398	0.515	0.519	1.414	1.370
ψ_{33}^x	0.478	0.460	1.414	1.436	0.479	0.461	1.414	1.449	0.479	0.460	1.414	1.500
ψ_{11}^y	0.504	0.500	1.414	1.403	0.502	0.515	1.414	1.437	0.502	0.525	1.414	1.440
ψ_{22}^y	0.535	0.548	1.414	1.457	0.535	0.534	1.414	1.440	0.536	0.547	1.414	1.383
ψ_{33}^y	0.465	0.459	1.414	1.461	0.466	0.470	1.414	1.388	0.465	0.476	1.414	1.400
ψ_{44}^y	0.559	0.552	1.414	1.417	0.561	0.562	1.414	1.404	0.560	0.564	1.414	1.420
ψ_{55}^y	0.422	0.426	1.414	1.416	0.420	0.415	1.414	1.433	0.422	0.412	1.414	1.382
ψ_{66}^y	0.479	0.463	1.414	1.394	0.481	0.471	1.414	1.397	0.481	0.490	1.414	1.416
ν_1	0.353	0.358	0.390	0.392	0.358	0.352	0.395	0.388	0.358	0.359	0.395	0.398
$\tau_{\theta_1}^2$	9.546	9.292	100.966	87.442	9.474	9.361	96.145	89.125	9.443	9.139	94.148	93.435

Note: ave_s= average SNR across 1000 replications under COV; ave_r= average SNR across 1000 replications under CORR; emp_s= empirical SNR by the values of $\hat{\theta}$ across 1000 replications under COV; emp_r= empirical SNR by the values of $\hat{\theta}_*$ across 1000 replications under CORR. The notations ψ_{jj}^x and ψ_{jj}^y under θ are measurement-error variances for COV and standard deviations of the manifest variables for CORR. The parameter ν_1 is the indirect effect of ξ_1 on η_2. The values following $\tau_{\theta_1}^2$ are the squared signal-to-noise ratios for parameters without measurement errors or standard deviations.

replications, we obtain 1000 values of parameter estimates and SNRs for each θ and θ_*, respectively. These 1000 SNR values are averaged and termed as the averaged SNR in the following presentation. With the 1000 values of $\hat{\theta}$, $\hat{\theta}_*$, $\hat{\nu}_1$ or $\hat{\nu}_{*1}$, we also obtained their sample means and sample standard deviations, denoted as $\bar{\theta}$ and s^2, respectively. The empirical SNR for the NML estimate of an individual parameter is then obtained by

$$\tilde{\tau} = \bar{\theta}/(N^{1/2}s).$$

For the 12 shared parameters in $\hat{\boldsymbol{\theta}}_1$ or $\hat{\boldsymbol{\theta}}_{*1}$, we also obtained the vector $\bar{\boldsymbol{\theta}}_1$ of sample means and the sample covariance matrix $\mathbf{S}_1$ via the 1000 replicated values. The multivariate empirical SNR is computed as

$$\tilde{\tau}_1^2 = \bar{\boldsymbol{\theta}}_1' \mathbf{S}_1^{-1} \bar{\boldsymbol{\theta}}_1 / N.$$

Let ave_s, emp_s, ave_r, and emp_r denote the average and empirical SNRs for parameter estimates across the $N_r =$ 1000 replications. Table 2 contains the values of the average and empirical SNRs for the 21 individual parameters, the indirect effect ν_1, and the multivariate SNR τ_1^2. The results in Table 2 indicate that the values of the empirical SNRs are closely approximated by the average SNRs, especially at $N =$ 1000. Whereas our current interest is in the advantage of the CORR parameterization over its COV counterpart.

It follows from Table 2 that the values of the averaged SNR under CORR (ave_r) are uniformly greater than their COV counterparts (ave_s) across all the parameters and the three conditions of N. The value of the ratio $\text{ave}_r/\text{ave}_s$ for the individual parameters ranges from 1.04 to 1.78. For the 12 shared parameters, the averaged multivariate SNR under CORR is about 10 times its COV counterpart. The same patterns also hold in Table 2 for the empirical SNRs between the two parameterizations.

In summary, the Monte Carlo results in this section agree well with the meta results presented in the previous section.

5. Conclusion and Discussion

In social and behavioral sciences theoretical constructs are commonly measured by indicators that contain errors. By effectively separating the true scores from measurement errors, SEM is the method for modeling the relationship among the latent constructs. However, we are not aware of any existing study comparing the statistical advantages between correlation structure model and its covariance counterpart. By parameterizing the LISREL model as a correlation structure model, this article showed that LISREL-correlation yields uniformly more efficient estimates than LISREL-covariance for essentially all parameters that are of substantive interest. Less errors in estimates imply greater statistical power in conducting the conventional NHT or other types of statistical tests (e.g., Wang et al., 2024). In addition, for normally distributed data, there is no need to have the sample standard deviations of the observed variables included in the analysis of correlation structure models. Thus, correlation structure models are preferred than covariance structure models for better fundamental properties of statistical modeling in addition to conforming with the convention of formulating psychological theories by correlations.

While correlation structure offers more efficient parameter estimates, many z-statistics under CORR in Tables A1 to A18 are still less than 1.96. We would like to point out a few facts on such a phenomenon: (1) 1.96 is the cutoff point (critical value) associated with a statistical significance test at level $\alpha = .05$, which is just a convention rather than justified by a scientific rationale over another level, say $\alpha = .10$. (2) Many factors contribute to the value of a z-statistic. A value of z less than 1.96 does not mean that the value of the parameter can be regarded as zero or must be tiny. (3) An alternative approach is to combine the conventional NHT with equivalence testing (ET) and minimal-effect testing (MET) (Counsell & Cribbie, 2015; Lakens et al., 2018; Murphy & Myors, 1999; Wang et al., 2024), which convey more information about the sizes of parameters than the conventional NHT. LISREL-correlation for conducting ET and MET is also expected to be more powerful than LISREL-covariance due to their differences in efficiency of parameter estimates.

In this article, all the parameters are estimated by NML and so are the corresponding SEs. These quantities can also be estimated by other methods (e.g., least squares for $\hat{\theta}$ and bootstrap for SEs). Actually, robust methods were involved in computing the results in Tables A12 and A13, where smaller weights were assigned to cases that locate farther away from the center of the observations. In particular, when the distribution of the sample has heavier tails than that of a normal distribution, robust methods can yield much more efficient estimates than NML (Yuan et al., 2004; Yuan & Gomer, 2021). In addition, instead of estimating the SEs via the Fisher information matrix associated with NML, we can estimate the SEs by the sandwich-type covariance matrix or the bootstrap/Monte Carlo methods. Also, we did not study the distribution properties of $\hat{\boldsymbol{\theta}}$ or $\hat{\boldsymbol{\theta}}_*$ nor the coverage rates of their corresponding conventional confidence intervals (CI). While additional studies are needed in these directions, we suspect that parameter estimates by LISREL-correlation are still more efficient than their LISREL-covariance counterparts when the SNRs are evaluated according to the SDs computed using either sandwich-type matrix or the bootstrap covariance matrix. Based on the findings in Falk (2018), we suspect that the bootstrap percentile CI is preferred for parameter inference of correlation structural models, especially for indirect effects.

Bentler (2007, p. 777) listed four possible options for conducting correlation structure analysis, and the NML method studied in this article corresponds to Method 3 while EQS has implemented Method 4 of the list. According to Bentler (2006, p. 143), the ML method for correlation structure analysis in EQS is essentially a normal-distribution-based generalized least squares (NGLS) method. This method was originally formulated by Jennrich (1970) and the weight matrix in the NGLS method (see Eqs. 5.62 and 5.63 of Bentler, 2006) can be the sample product-moment correlation matrix or the model-implied correlation matrix (Shapiro & Browne, 1990). Since NGLS and NML are asymptotically equivalent for correctly specified models in covariance structure analysis (Browne, 1974), we suspect that the NGLS method for correlation structure analysis also outperforms its counterpart for covariance structure analysis with respect to parameter efficiency as measured by SNRs. But additional study is needed in this direction.

The second method listed by Bentler (2007) is standardized solutions following a covariance structure analysis, and he stated "This is a typical practice. It is acceptable only with scale-invariant models. Also, standard errors and z statistics for the standardized estimates that generate $\hat{\mathbf{P}}$ should be used. They are known (Jamshidian & Bentler, 2000) but not generally available." The "should be" standard errors by Bentler refer to those that are obtained by the delta method. Note that parameter estimates $\hat{\boldsymbol{\theta}}_{*1}$ under LISREL-correlation are not necessarily standardized solutions. For a scale-invariant confirmatory factor model when all the latent factors are anchored by $\phi = \mathrm{Var}(\xi) = 1.0$, standardized factor loadings following a covariance structure analysis should be mathematically identical to the factor loading under correlation structure analysis presented in this article. For parameters other than factor loading of a CFA model, even if their standardized solutions are different from the corresponding estimates of correlation structure analysis, the two sets of z-statistics or SNRs should be mathematically identical if the model is scale-invariant and the SEs of standardized solutions are computed by the standard delta method. Jamshidian and Bentler (2000) also described a higher-order approximation for carrying out the delta-method. Then the SEs for standardized solutions by such an approximation will not be equivalent to those under NML-based correlation structure modeling and neither the z-statistics or SNRs.

We did not elaborate on goodness of fit of the models between the two parameterizations in this article. This is because LISREL-correlation and LISREL-covariance are equivalent in the overall model structure whenever the model is scale invariant. Thus, all fit indices developed for covariance structure models equally apply to correlation structure models. In particular, for the measure SRMR (standardized root mean squared residual), we can compute the standardized residuals by $\mathbf{R} - \mathbf{P}(\hat{\boldsymbol{\theta}}_{*1})$ instead of rescaling the residuals of the covariance structure model. For models that are not-scale-invariant, we can endorse the parameterization that has a significantly better fit. For example, we can endorse the correlation structure model over the covariance structure model or the other way around when $T_{ml} - T_{ml*}$ is greater or smaller than 2 times the SE of the difference of the two test statistics, where the subscript $*$ in T_{ml} is for the likelihood ratio statistic by LISREL-correlation and T_{ml} is for LISREL-covariance. More studies are needed in this direction.

We did not discuss models with mean structures in this article. This is because the rationale for standardizing variables and/or correlation structure analysis does not get along with that for studying mean changes. In contrast, correlation structure analysis is the principle for modeling ordinal data, where a multivariate normal distribution is assumed underlying the observed frequencies. While there exist categorical differences between the analysis of a polychoric correlation matrix and the NML method for correlation structure analysis studied in this article, the two modeling scenarios are connected (see Bentler, 2006, pp. 140–149). Readers are referred to Christoffersson (1975), Muthén (1978), and Bentler and Savalei (2010) for elements of developments under each scenario.

While EQS has implemented the NGLS method for correlation structure analysis, software development for correlation structure modeling is far behind its covariance counterpart. See the reviews by Kline (1998) and Narayanan (2012) where correlation structure analysis was not even mentioned. To our knowledge, no software program has the option for NML correlation structure analysis, as studied in this article. For programs that allow users to specify nonlinear constraints (e.g., LISREL), one might be able to force the program to conduct NML correlation structure analysis by implementing proper constraints (see e.g., Bentler & Lee, 1983; Jamshidian & Bentler, 1993; Lee & Poon, 1985). However, with the increase of number of variables, specifying constraints or using phantom variables can be cumbersome (e.g., Chan, 2007; Kwan & Chan, 2011; Rindskopf, 1984). Such techniques might only apply to simple models and by researchers who are familiar with the involved equations as well as the software/program. In addition, most SEM packages contain the output of standardized solutions, and lavaan (Rosseel, 2012) has the option of outputting standard errors for standardized solutions. There also exist R packages for computing indirect effects and their SEs with standardized variables by the bootstrap methodology (e.g., Cheung & Cheung, 2024). However, standardized solutions are a postdoc manipulation on parameter estimates of covariance structure analysis, which are different from the results of correlation structure analysis, as noted by Bentler (2007) and clarified in this article. We would hope that the fundamental statistical advantages of correlation structure models showed in this article would stimulate the development for general SEM software to conduct its analysis. In this direction, we would like to note that the Bentler-Weeks model (Bentler & Weeks, 1980) and the RAM model (McArdle, 2005; McArdle & McDonald, 1984) can equally serve as the platform for correlation structure analysis although the LISREL model is used in the current study.

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